

Russian Research Center "Kurchatov Institute"

A.L.Barabanov¹

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MODEL FOR RESONANCE ENHANCEMENT
OF P- AND T-NONINVARIANT EFFECTS
IN NEUTRON REACTIONS

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¹e-mail: x1052@kiae.su

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We consider a simplified model for resonant neutron-nucleus interaction with coupled channels. An analytical solution is given for two coupled channels and arbitrary neutron orbital momentum. A case of a weak channel coupling, corresponding to narrow Breit-Wigner resonance, is analyzed in details. As far as the total width of a resonance coincides with the neutron width, the model is directly appropriate only for light nuclei. We study a mixing of two narrow s- and p-wave resonances by P- and P-,T-noninvariant potentials in the first order of perturbation theory. As an example a close-lying pair of s- and p-wave resonances of the ^{35}Cl nucleus is considered. In a resonance with an orbital momentum l an enhancement of mixing amplitude in comparison with potential scattering is $\sim (\omega/\gamma_l)^{1/2}(1/(kR)^{2l+1})$, where ω is a characteristic single-particle energy and γ_l is a reduced width of a resonance. The favorable possibilities are shown to exist on thick targets for measurements beyond the resonance widths. In particular, an interference minimum near s-wave resonance is of interest for P-odd neutron spin rotation on light spinless nuclei.

1 Introduction

The question of scale and nature of T-invariance violation in fundamental interactions remains open. So far only the results, obtained in K^0 -meson decays [1], evidence for such violation. Many researches of T-noninvariant effects in nuclear processes, in neutron β -decay, as well as studies of electric dipole moments of elementary particles have lowered the upper limit on mixing of T-noninvariant forces to T-invariant ones to the level of $10^{-3} - 10^{-4}$ (see, e.g., Ref.[2]). Now possible T-noninvariant effects in isolated compound-resonances are widely discussed [3]-[10]. An interest to them results from huge enhancement of P-noninvariant effects. As it was shown for the first time in Ref.[11], transmission asymmetry of neutrons with opposite helicities in p-wave resonances may be as much as $10^{-3} - 10^{-1}$ that is 5-7 order greater than a similar effect in N-N scattering. The review of recent developments in the study of P-invariance violation in neutron resonances is given in Ref.[12]. Now a sign correlation of P-odd effects, measured in several resonances of the ^{232}Th nucleus [13], attracts particular attention. This result is discussed, e.g., in Refs.[14]-[18].

Usually one distinguishes dynamic and structural (or kinematic) enhancements. The first is due to high density of resonances [19]-[21]. As it was shown in Refs.[22]-[24], this enhancement should take place for T-noninvariant effects as well as for P-noninvariant ones. A structural enhancement [25] (see, also, [26]) arises in p-wave resonances, while in s-wave ones we have a structural suppression. Both type of enhancement do not represent the fact that the observables peak in resonances. So a concept of resonance enhancement was brought into practice in Refs.[27, 28], where an energy dependence of P- and T-noninvariant effects near s- and p-wave resonances has been investigated. This type of enhancement results from an increase of time spent by a neutron in the weak-interaction field of a target. It was first mentioned in Refs.[29, 30].

As far as a structure of highly excited nuclei is very complex, there is no consistent microscopic theory of neutron resonances. Thus phenomenological models as R-matrix theory [31] or shell model approach to nuclear reactions [32] are used to estimate the observables. There exist, however, some uncertainties in these models in the phases of the S-matrix elements, caused by potential scattering or contributions of distant resonances. This may lead to uncontrolled fake effects, which make difficult a search of T-invariance violation (see, e.g., Ref.[4]).

In this paper we study P- and T-noninvariant effects in simplified, but exactly solvable model of neutron-nucleus interaction. The model reproduces narrow resonances, occurring owing to excitations of selected degrees of freedom of a target. The basis for the model is a scheme of coupled channels [33], described in section 2. In section 3 P- and T-noninvariant observables are expressed in terms of S-matrix elements. A method for calculation of P- and T-noninvariant corrections to S-matrix is presented in section 4. The exactly solvable model

of resonant neutron-nucleus interaction is stated in section 5. In section 6 this model is used for description of a close-lying pair of s- and p-wave resonances and their mixing by P- and T-noninvariant interactions. The results of illustrative calculation of P- and T-noninvariant observables are presented in section 7. In section 8 a summary of the most important conclusions is given.

2 Scheme of coupled channels

We consider an interaction of a neutron with a nucleus of mass A . Let \hat{H}_A is a nuclear hamiltonian, and ε_α and $\psi_\alpha(\tau)$ are its eigenvalues and orthonormal eigenfunctions

$$\hat{H}_A \psi_\alpha(\tau) = \varepsilon_\alpha \psi_\alpha(\tau), \quad \langle \psi_{\alpha'} | \psi_\alpha \rangle = \delta_{\alpha'\alpha}, \quad (1)$$

τ is a set of internal variables. The hamiltonian \hat{H}_A will be considered as invariant with respect to rotations and space and time inversions (R-, P- and T-invariant, respectively). We assume that the nuclear spectrum is purely discrete. An index α includes spin I , its projection μ on an axis z , parity π and number i , which distinguishes states with the same I , μ and π . An energy ε_α does not depend on μ because of R-invariance of hamiltonian \hat{H}_A

In a center-of-mass system of neutron and nucleus a total hamiltonian is of the form

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \mathbf{r}^2} + \hat{U} + \hat{H}_A, \quad (2)$$

where $\mathbf{r} = \mathbf{r}_n - \mathbf{R}_A$ is a neutron radius-vector with respect to a target center of mass, m is a reduced mass, and \hat{U} is an operator of interaction of a neutron with target nucleons.

Let us introduce the orthonormal single-particle spin-angular functions

$$|j\nu; l\rangle = \sum_{m\sigma} C_{lms\sigma}^{j\nu} i^l Y_{lm}(\mathbf{r}) \chi_{s\sigma}, \quad \langle j'\nu'; l' | j\nu; l \rangle = \delta_{j'\nu'} \delta_{\nu'\nu} \delta_{l'l}. \quad (3)$$

Here $\chi_{s\sigma}$ is a spinor, describing a state of a neutron with a projection σ of a spin $s = 1/2$ on an axis z , $Y_{lm}(\mathbf{r})$ are spherical harmonics, l and m are relative neutron-nucleus orbital momentum and its projection on an axis z , and $C_{lms\sigma}^{j\nu}$ are Clebsh-Gordan coefficients. Angular momentum $\mathbf{j} = \mathbf{l} + \mathbf{s}$ will be named a neutron total angular momentum. Functions

$$|JM; lj\alpha\rangle = \sum_{\nu\mu} C_{j\nu l\mu}^{JM} |j\nu; l\rangle \psi_\alpha(\tau) \quad (4)$$

are also orthonormal

$$\langle J'M'; l'j'\alpha' | JM; lj\alpha \rangle = \delta_{J'J} \delta_{M'M} \delta_{l'l} \delta_{j'j} \delta_{\alpha'\alpha}. \quad (5)$$

Here J and M are a total angular momentum and its projection on an axis z . The phases are so chosen that the wave functions have usual transform properties with respect to time inversion [34]

$$\begin{aligned}\hat{T}\psi_{I\mu}(\tau) &= (-1)^{I+\mu}\psi_{I-\mu}(\tau), & \hat{T}\chi_{s\sigma} &= (-1)^{s+\sigma}\chi_{s-\sigma}, \\ \hat{T}|j\nu; l > &= (-1)^{j+\nu}|j - \nu; l >, & \hat{T}|JM; lj\alpha > &= (-1)^{J+M}|J - M; lj\alpha > .\end{aligned}\quad (6)$$

Let us present a total wave function of neutron and nucleus as a series in state vectors (4)

$$\Psi(\mathbf{r}, \tau) = \sum_{JM} \sum_{lj\alpha} R_{ljJM}^{(\alpha)}(r) |JM; lj\alpha > . \quad (7)$$

Substituting this expansion in the Schrödinger equation

$$\hat{H}\Psi(\mathbf{r}, \tau) = E\Psi(\mathbf{r}, \tau), \quad (8)$$

and projecting on $\langle JM; lj\alpha |$, we get

$$\begin{aligned}\frac{d^2 R_{ljJM}^{(\alpha)}(r)}{dr^2} + \frac{2}{r} \frac{dR_{ljJM}^{(\alpha)}(r)}{dr} - \frac{l(l+1)}{r^2} R_{ljJM}^{(\alpha)}(r) + \frac{2m(E - \varepsilon_\alpha)}{\hbar^2} R_{ljJM}^{(\alpha)}(r) - \\ - \frac{2m}{\hbar^2} \sum_{l'j'\alpha'} \langle JM; lj\alpha | \hat{U} | JM; l'j'\alpha' > R_{l'j'JM}^{(\alpha')}(r) = 0.\end{aligned}\quad (9)$$

The matrix element of an operator \hat{U} is diagonal on J and M and does not depend on M because of R-invariance of interaction.

We assume that the interaction \hat{U} is short-range and vanishes at $r > R$. Then outside the interaction region the total wave function is an eigenfunction of hamiltonian of free motion. So it can be presented as an expansion in spherical Hankel functions $h_l^{(\pm)}(x)$

$$\begin{aligned}\Psi_{\alpha_0}(\mathbf{r}, \tau)|_{r>R} &= \sum_{JM} \sum_{lj} a(ljJM) \left(h_l^{(-)}(k_{\alpha_0} r) |JM; lj\alpha_0 > + \right. \\ &\quad \left. + \sum_{l'j'} S_J(lj\alpha_0 \rightarrow l'j'\alpha_0) h_{l'}^{(+)}(k_{\alpha_0} r) |JM; l'j'\alpha_0 > \right) + \\ &\quad + \sum_{\alpha \neq \alpha_0} \left(\frac{k_\alpha}{k_{\alpha_0}} \right)^{1/2} \sum_{JM} \sum_{lj} a(ljJM) \sum_{l'j'} S_J(lj\alpha_0 \rightarrow l'j'\alpha) h_{l'}^{(+)}(k_\alpha r) |JM; l'j'\alpha > .\end{aligned}\quad (10)$$

This function describes a neutron scattering by nucleus being in the state α_0 . A relative momentum, $\hbar k_{\alpha_0}$, in an entrance channel is given by equation

$$E = \frac{(\hbar k_{\alpha_0})^2}{2m} + \varepsilon_{\alpha_0}. \quad (11)$$

Elements of S-matrix, $S_J(lj\alpha_0 \rightarrow l'j'\alpha_0)$, correspond to elastic scattering with a total angular momentum J . Orbital, l , and total, j , neutron angular momenta can change in elastic scattering within the limits of the rules of angular momentum summation ($|l-s| \leq j \leq l+s$, $|I-j| \leq J \leq I+j$). Nuclear transition to a state $\alpha \neq \alpha_0$ corresponds to inelastic scattering. A relative momentum in an inelastic channel is

$$\hbar k_\alpha = (2m(E - \varepsilon_\alpha))^{1/2}. \quad (12)$$

If $\varepsilon_\alpha > E$, then $\hbar k_\alpha = i(\hbar q_\alpha)$, where q_α is a real positive quantity. Taking into account an asymptotic of Hankel functions

$$h_l^{(\pm)}(x) \xrightarrow{x \rightarrow +\infty} (\mp i)^{l+1} \frac{\exp(\pm ix)}{x}, \quad (13)$$

we see, that the channel with the energy $\varepsilon_\alpha > E$ is closed (the wave function falls off exponentially at long range). The function (10) contains factors

$$a(ljJM) = 2\pi \sum_{\nu\mu m\sigma} C_{j\nu I\mu}^{JM} C_{lms\sigma}^{j\nu} a_\mu(I) a_\sigma(s) Y_{lm}^*(\mathbf{k}_{\alpha_0}), \quad (14)$$

where $a_\mu(I)$ and $a_\sigma(s)$ are amplitudes of nucleus and neutron states with spin projections μ and σ on an axis z , respectively.

Comparing a general expansion (7) for the total wave function with its form (10) outside the interaction region, we find for radial functions

$$\begin{aligned} R_{ljJM}^{(\alpha)}(r)|_{r>R} &= \left(\frac{k_\alpha}{k_{\alpha_0}}\right)^{1/2} \sum_{l_0 j_0} a(l_0 j_0 JM) \left(\delta_{ll_0} \delta_{jj_0} \delta_{\alpha\alpha_0} h_l^{(-)}(k_\alpha r) + \right. \\ &\quad \left. + S_J(l_0 j_0 \alpha_0 \rightarrow lj\alpha) h_l^{(+)}(k_\alpha r) \right). \end{aligned} \quad (15)$$

Let us write these radial functions in the form

$$R_{ljJM}^{(\alpha)}(r) = 2 \sum_{l_0 j_0} a(l_0 j_0 JM) \frac{F_{n_0 n}^J(r)}{r}, \quad (16)$$

where an index n includes l , j and α . An index n_0 suggests, that the functions $F_{n_0 n}^J(r)$ describe scattering of a neutron with initial orbital, l_0 , and total, j_0 , angular momenta on a nucleus, being in a state α_0 . Substituting the functions (16) in the equations (9) and separating the terms at the same factors $a(l_0 j_0 JM)$, we get

$$\frac{d^2 F_{n_0 n}^J(r)}{dr^2} - \sum_{n'} r < JM; n | \frac{2m\hat{U}}{\hbar^2} + \frac{l(l+1)}{r^2} - k_n^2 | JM; n' > \frac{F_{n_0 n'}^J(r)}{r} = 0. \quad (17)$$

Outside the interaction region the functions $F_{n_0 n}^J(r)$ take the form

$$F_{n_0 n}^J(r)|_{r>R} = \frac{1}{2(k_{n_0}k_n)^{1/2}} \left(\delta_{nn_0} k_n r h_l^{(-)}(k_n r) + S_J(n_0 \rightarrow n) k_n r h_l^{(+)}(k_n r) \right). \quad (18)$$

We assume that the interaction has no singularity at the point $r = 0$. Then the radial functions should be regular at the origin

$$F_{n_0 n}^J(0) = 0. \quad (19)$$

Thus, in the scheme of coupled channels, that we have considered, the set of equations (17) completely determines the dynamics of nuclear reaction. Solving these equations with boundary conditions (18), (19), we get the S-matrix elements and, consequently, all observables.

3 P- and T-noninvariant observables

In neutron transmission experiments one studies asymmetry of a total cross section and spin rotation, both caused by parity nonconservation or possible T-invariance violation. The observables may be expressed in terms of a forward elastic scattering amplitude. This amplitude, averaged over spin states of nuclei, is a matrix 2×2 in a neutron spin space. Thus it can be presented as an expansion in Pauli matrixes σ_i

$$f(0) = F_0 + (\boldsymbol{\sigma} \mathbf{n}) F_1, \quad (20)$$

where \mathbf{n} is some unit vector.

According to optical theorem a total cross section is

$$\sigma_t = \frac{4\pi}{k} (\text{Im} F_0 + p_1(s)(\mathbf{n}_s \mathbf{n}) \text{Im} F_1). \quad (21)$$

Here $p_1(s)$ is a neutron polarization, and \mathbf{n}_s is a unit vector along an axis of polarization. We see, that an asymmetry of a total cross section for neutrons, polarized along a direction \mathbf{n} or opposite to it, is expressed in terms of imaginary part of factor F_1 . On the other hand, using the methods of neutron optics [35], we obtain for polarization of neutrons, passed through a target of density ρ and thickness d

$$p'_1(s) \mathbf{n}'_s = p_1(s) \mathbf{n}_s + \frac{4\pi n}{k} p_1(s) [\mathbf{n}_s \mathbf{n}] \text{Re} F_1 - \frac{4\pi n}{k} \mathbf{n} \text{Im} F_1, \quad (22)$$

where $n = \rho d$ is a number of nuclei on a unit of target area. Clearly, an angle of spin rotation around a vector \mathbf{n} is determined by real part of factor F_1 . The last term in Eq.(22) describes polarization of passed neutrons, arising owing to asymmetry of a total cross section.

If target nuclei are not oriented, a vector \mathbf{n} in Eq.(20) coincides with a unit vector \mathbf{n}_k along a momentum $\hbar\mathbf{k}$ of incident neutrons. A relevant P-noninvariant correlation ($\boldsymbol{\sigma}\mathbf{n}_k$) was considered for the first time in Ref.[36]. As noted in introduction, this correlation was being studied intensively last years. It was pointed out in Refs.[37, 38] that nuclear polarization makes possible the study of P- and T-noninvariant correlation ($\boldsymbol{\sigma}[\mathbf{n}_k\mathbf{n}_I]$), where \mathbf{n}_I is a unit vector along an axis of nuclear orientation. It was shown in Refs.[39]-[41] that T-noninvariant, but P-even correlation ($\boldsymbol{\sigma}[\mathbf{n}_k\mathbf{n}_I])(\mathbf{n}_k\mathbf{n}_I)$) can be studied on an aligned target. In this paper we restrict our attention to P- and P-,T-noninvariant effects, so we shall consider only a polarized target.

Let $p_1(I)$ is nuclear polarization. Taking into account only s- and p-waves, we express the factors F_0 and F_1 in terms of the S-matrix elements, $S_J(lj \rightarrow l'j')$, corresponding to elastic scattering (we omit everywhere in this section an elastic-channel index α_0)

$$F_0 = a_0 + b_2 p_1(I)(\mathbf{n}_k\mathbf{n}_I), \quad (23)$$

$$F_1 \mathbf{n} = a_1 p_1(I)\mathbf{n}_I + a_2 p_1(I)(3\mathbf{n}_k(\mathbf{n}_I\mathbf{n}_k) - \mathbf{n}_I) + b_1 \mathbf{n}_k + c_1 p_1(I)[\mathbf{n}_k\mathbf{n}_I]. \quad (24)$$

A coefficient a_0 takes the form

$$a_0 = \frac{i}{2k} \sum_J g_J \sum_{lj} (1 - S_J(lj \rightarrow lj)), \quad (25)$$

where $g_J = (2J+1)/2(2I+1)$ is a statistical factor. Coefficients a_1 and a_2 specify spin-spin interaction of a neutron and nucleus

$$a_1 = \frac{i}{2k} \sum_J \left(A_J^{(1)} (1 - S_J(0\frac{1}{2} \rightarrow 0\frac{1}{2})) + \sum_{jj'} A_{Jjj'}^{(2)} (\delta_{jj'} - S_J(1j \rightarrow 1j')) \right), \quad (26)$$

$$a_2 = \frac{i}{2k} \sum_{Jjj'} A_{Jjj'}^{(3)} (\delta_{jj'} - S_J(1j \rightarrow 1j')). \quad (27)$$

Explicit expressions for numerical factors A , as well as for factors B and C , defined below, are presented in an appendix. Coefficients b_1 and b_2 for P-odd correlations are given by formulas

$$b_1 = \frac{i}{2k} \sum_J g_J (S_J(0\frac{1}{2} \rightarrow 1\frac{1}{2}) + S_J(1\frac{1}{2} \rightarrow 0\frac{1}{2})), \quad (28)$$

$$b_2 = \frac{i}{2k} \sum_{Jj} B_{Jj} (S_J(0\frac{1}{2} \rightarrow 1j) + S_J(1j \rightarrow 0\frac{1}{2})). \quad (29)$$

Finally, we have for magnitude of possible P-,T-noninvariant effect

$$c_1 = \frac{1}{2k} \sum_{Jj} C_{Jj} (S_J(0\frac{1}{2} \rightarrow 1j) - S_J(1j \rightarrow 0\frac{1}{2})). \quad (30)$$

We see, the coefficients b_1 , b_2 and c_1 are due to transitions from s- to p-wave and vice versa, that is possible only at parity nonconservation. While the coefficient c_1 differs from zero, if additionally a symmetry of S-matrix with respect to the main diagonal is violated.

If a target is not polarized, then $p_1(I) = 0$, therefore $F_0 = a_0$, $F_1 = b_1$, $\mathbf{n} = \mathbf{n}_k$. Thus, as noted, the unique correlation $(\sigma \mathbf{n}_k)$ remains. Let σ_+ and σ_- are total cross section for neutrons, completely polarized along the vector \mathbf{n}_k and opposite to it, respectively. Thus, we obtain for P-noninvariant asymmetry of a total cross section

$$\Delta\sigma_P \equiv \sigma_+ - \sigma_- = \frac{4\pi}{k^2} \sum_J g_J \text{Re}(S_J(0\frac{1}{2} \rightarrow 1\frac{1}{2}) + S_J(1\frac{1}{2} \rightarrow 0\frac{1}{2})). \quad (31)$$

At the same time according to Eq.(22) the angle of spin rotation for transversely polarized neutrons, counted in a direction of right-hand rotation along the vector \mathbf{n}_k , equals

$$\chi_P = \frac{2\pi n}{k^2} \sum_J g_J \text{Im}(S_J(0\frac{1}{2} \rightarrow 1\frac{1}{2}) + S_J(1\frac{1}{2} \rightarrow 0\frac{1}{2})). \quad (32)$$

We turn now to the case, when target nuclei are polarized transversely the neutron momentum ($\mathbf{n}_I \perp \mathbf{n}_k$). Let σ_\uparrow and σ_\downarrow are total cross section for neutrons, completely polarized along the vector $[\mathbf{n}_k \mathbf{n}_I]$ and opposite to it, respectively. Then we have for P-,T-noninvariant asymmetry of a total cross section

$$\Delta\sigma_{PT} \equiv \sigma_\uparrow - \sigma_\downarrow = \frac{4\pi}{k^2} \sum_J C_{Jj} \text{Im}(S_J(0\frac{1}{2} \rightarrow 1j) - S_J(1j \rightarrow 0\frac{1}{2})). \quad (33)$$

A contribution of P-,T-noninvariant correlation to the angle of spin rotation around the vector $[\mathbf{n}_k \mathbf{n}_I]$ for neutrons, initially polarized along \mathbf{n}_I , is

$$\chi_{PT} = -\frac{2\pi n}{k^2} \sum_J C_{Jj} \text{Re}(S_J(0\frac{1}{2} \rightarrow 1j) - S_J(1j \rightarrow 0\frac{1}{2})). \quad (34)$$

In reality spin-spin forces, as well as P-odd correlation $(\sigma \mathbf{n}_k)$ make difficult a measurement of the asymmetry (33) and the angle (34). Now the ways of suppression of the fake effects are intensively discussed [42]-[44].

In transmission experiments an asymmetry

$$\epsilon = \frac{N^+ - N^-}{N^+ + N^-}, \quad (35)$$

is directly measured, where N^+ and N^- are numbers of neutrons, passed through a target for two opposite values of their initial polarization. It is easy to show (see, e.g., Ref.[11]) that these observable asymmetries express by asymmetries of a total cross section in the following way

$$\epsilon_{P(PT)} = -n \frac{\Delta\sigma_{P(PT)}}{2}. \quad (36)$$

4 First order corrections to S-matrix

Weak interaction, violating P-invariance and, possibly, T-invariance, can be accounted in the scheme of coupled channels as perturbation. We separate from the operator of neutron-nucleon interaction a small component $\delta\hat{U}$

$$\hat{U} \longrightarrow \hat{U} + \delta\hat{U}. \quad (37)$$

We assume that the operator \hat{U} on the right side of Eq.(37) is T-invariant.

A perturbation $\delta\hat{U}$ results in small change of radial functions and S-matrix elements

$$F_{n_0 n}^J(r) \longrightarrow F_{n_0 n}^J(r) + \delta F_{n_0 n}^J(r), \quad S_J(n_0 \rightarrow n) \longrightarrow S_J(n_0 \rightarrow n) + \delta S_J(n_0 \rightarrow n). \quad (38)$$

Substituting Eqs.(37), (38) to Eqs.(17), (18), we get in zero order in $\delta\hat{U}$ a homogeneous set of equations for functions $F_{n_0 n}^J(r)$

$$\frac{d^2 F_{n_0 n}^J(r)}{dr^2} - \sum_{n'} q_{nn'}^J(r) F_{n_0 n'}^J(r) = 0 \quad (39)$$

with boundary conditions (18) and (19). S-matrix, determined by T-invariant interaction \hat{U} , is symmetric with respect to the main diagonal

$$S_J(n_0 \rightarrow n) = S_J(n \rightarrow n_0). \quad (40)$$

We assume that the matrix element of the operator of strong interaction, \hat{U} , between states $|JM; n\rangle$ is a function, dependent on r . So we obtain

$$q_{nn'}^J(r) = \langle JM; n | \frac{2m\hat{U}}{\hbar^2} + \frac{l(l+1)}{r^2} - k_n^2 | JM; n' \rangle. \quad (41)$$

This matrix is real and symmetric

$$q_{nn'}^J(r) = q_{n'n}^{J*}(r) = q_{n'n}^J(r) \quad (42)$$

due to hermiticity, R- and T-invariance of the operator \hat{U} [34].

In first order in $\delta\hat{U}$ a non-homogeneous set of equations for functions $\delta F_{n_0 n}^J(r)$ arises

$$\frac{d^2 \delta F_{n_0 n}^J(r)}{dr^2} - \sum_{n'} q_{nn'}^J(r) \delta F_{n_0 n'}^J(r) = Q_{n_0 n}^J(r) \quad (43)$$

with boundary conditions

$$\delta F_{n_0 n}^J(r)|_{r>R} = \frac{1}{2(k_{n_0} k_n)^{1/2}} \delta S_J(n_0 \rightarrow n) k_n r h_l^{(+)}(k_n r), \quad (44)$$

$$\delta F_{n_0 n}^J(0) = 0. \quad (45)$$

Sources on the right side of Eqs.(43) are of the form

$$Q_{n_0 n}^J(r) = r \sum_{n'} \langle JM; n | \frac{2m \delta \hat{U}}{\hbar^2} | JM; n' \rangle \frac{F_{n_0 n'}^J(r)}{r}. \quad (46)$$

We shall search solutions of the non-homogeneous set of equations (43) as series in linearly independent solutions of the homogeneous set of equations (39). We accept for definiteness, that the index n takes on values of $1, 2 \dots N$. It is known, that a set of N differential equations of second order has $2N$ linearly independent solutions. For each value n_0 the regular solution $F_{n_0 n}^J(r)$ of the set (39) exists. Besides for each n_0 one can construct an irregular solution, satisfying to condition

$$G_{n_0 n}^J(r)|_{r>R} = \delta_{n_0 n} k_n r h_l^{(+)}(k_n r). \quad (47)$$

As far as n_0 also takes on N values ($n_0 = 1, 2 \dots N$), the functions $F_{n_0 n}^J(r)$ and $G_{n_0 n}^J(r)$ form the required fundamental system of $2N$ linearly independent solutions. It is easy to show, that Wronskian determinant (strokes designate differentiation with respect to r)

$$W^J(r) = \begin{vmatrix} F_{11}^J(r) & F_{21}^J(r) & \dots & F_{N1}^J(r) & G_{11}^J(r) & \dots & G_{N1}^J(r) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ F_{1N}^J(r) & F_{2N}^J(r) & \dots & F_{NN}^J(r) & G_{1N}^J(r) & \dots & G_{NN}^J(r) \\ F_{11}^{J'}(r) & F_{21}^{J'}(r) & \dots & F_{N1}^{J'}(r) & G_{11}^{J'}(r) & \dots & G_{N1}^{J'}(r) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ F_{1N}^{J'}(r) & F_{2N}^{J'}(r) & \dots & F_{NN}^{J'}(r) & G_{1N}^{J'}(r) & \dots & G_{NN}^{J'}(r) \end{vmatrix}, \quad (48)$$

does not depend on r and equals i^N .

We write now the solutions of the non-homogeneous set of equations (43) in the form

$$\delta F_{n_0 n}^J(r) = \sum_{n'} \left(\int_0^r \beta_{n_0 n'}^J(r') dr' + b_{n_0 n'}^J \right) F_{n' n}^J(r) + \sum_{n'} \left(\int_0^r \gamma_{n_0 n'}^J(r') dr' + c_{n_0 n'}^J \right) G_{n' n}^J(r). \quad (49)$$

Substituting these functions to the set (43), we obtain $2N$ algebraic equations for $2N$ functions $\beta_{n_0 n}^J(r)$ and $\gamma_{n_0 n}^J(r)$ for each value of n_0

$$\begin{cases} \sum_{n'} \beta_{n_0 n'}^J(r) F_{n' n}^J(r) + \sum_{n'} \gamma_{n_0 n'}^J(r) G_{n' n}^J(r) = 0, \\ \sum_{n'} \beta_{n_0 n'}^J(r) F_{n' n}^{J'}(r) + \sum_{n'} \gamma_{n_0 n'}^J(r) G_{n' n}^{J'}(r) = Q_{n_0 n}^J(r). \end{cases} \quad (50)$$

This set is tractable because of $W^J = i^N$ is not equal to zero.

Constants $b_{n_0n}^J$ and $c_{n_0n}^J$ can be found from the boundary conditions. Indeed, owing to regularity of the functions $\delta F_{n_0n}^J(r)$ in the origin we have

$$c_{n_0n}^J = 0. \quad (51)$$

On the other hand, as far as the functions $\delta F_{n_0n}^J(r)$ (44) outside the interaction region contain only diverging waves, coefficients at functions $F_{n'n}^J(r)$ should vanish at $r > R$. This gives

$$b_{n_0n}^J = - \int_0^R \beta_{n_0n}^J(r) dr. \quad (52)$$

We notice, that an upper limit in this integral may take any value greater than R , as at $r > R$ we have $Q_{n_0n}^J(r) = 0$ and, correspondingly, $\beta_{n_0n}^J(r) = \gamma_{n_0n}^J(r) = 0$.

So we get for the functions $\delta F_{n_0n}^J(r)$ outside the interaction region

$$\delta F_{n_0n}^J(r)|_{r>R} = \left(\int_0^\infty \gamma_{n_0n}^J(r) dr \right) k_n r h_l^{(+)}(k_n r). \quad (53)$$

Comparing this result with Eq.(44), we find the first order corrections to the S-matrix

$$\delta S_J(n_0 \rightarrow n) = 2(k_{n_0} k_n)^{1/2} \int_0^\infty \gamma_{n_0n}^J(r) dr. \quad (54)$$

We are interesting now in the functions $\gamma_{n_0n}^J(r)$. They can be written in the form

$$\gamma_{n_0n}^J(r) = \frac{1}{W^J} \sum_{n'} Q_{n_0n'}^J(r) A_{nn'}^J(r). \quad (55)$$

Here $A_{nn'}^J(r)$ is an algebraic adjoint of an element $G_{nn'}^{J'}(r)$ of the Wronskian determinant (48). It is easy to check, that the algebraic adjoints $A_{nn'}^J(r)$ satisfy a homogeneous set of equations

$$\frac{d^2 A_{nn'}^J(r)}{dr^2} - \sum_{n''} q_{n''n'}^J(r) A_{nn''}^J(r) = 0. \quad (56)$$

As far as the matrix $q_{n''n'}^J(r)$ is symmetric, the functions $A_{nn'}^J(r)$ are solutions of Eqs.(39). On the other hand, the adjoints $A_{nn'}^J(r)$ outside the interaction region take the form

$$A_{nn}^J(r)|_{r>R} = i^{N-1} \frac{1}{2k_n} \left(k_n r h_l^{(-)}(k_n r) + S_J(n \rightarrow n) k_n r h_l^{(+)}(k_n r) \right), \quad (57)$$

$$A_{nn'}^J(r)|_{r>R} = i^{N-1} \frac{1}{2(k_n k_{n'})^{1/2}} S_J(n' \rightarrow n) k_{n'} r h_l^{(+)}(k_{n'} r), \quad n \neq n'. \quad (58)$$

In view of the symmetry of the S-matrix (40), these functions $A_{nn'}^J(r)$ are proportional to the regular solutions $F_{nn'}^J(r)$ (18) at $r > R$. Thus, we conclude, that for any value of r a relationship holds

$$A_{nn'}^J(r) = i^{N-1} F_{nn'}^J(r), \quad (59)$$

because of the functions $A_{nn'}^J(r)$ and $F_{nn'}^J(r)$ are solutions of the same set of equations (39).

Substituting explicit expressions for the sources $Q_{n_0 n}^J(r)$ (46) and the algebraic adjoints $A_{nn'}^J(r)$ (59) to Eq.(55), we obtain the functions $\gamma_{n_0 n}^J(r)$. Thus the first order corrections to the S-matrix are of the form

$$\begin{aligned} \delta S_J(n_0 \rightarrow n) &= -2i(k_{n_0} k_n)^{1/2} \times \\ &\times \sum_{n' n''} \int_0^\infty r^2 dr \left(\frac{F_{nn'}^J(r)}{r} < JM; n' | \frac{2m\delta\hat{U}}{\hbar^2} | JM; n'' > \frac{F_{n_0 n''}^J(r)}{r} \right). \end{aligned} \quad (60)$$

If the interaction $\delta\hat{U}$ is T-invariant, the matrix element on the right side of Eq.(60) is symmetric, therefore we have

$$\delta S_J(n_0 \rightarrow n) = \delta S_J(n \rightarrow n_0). \quad (61)$$

While if $\delta\hat{U}$ anticommutes with the operator of time inversion, \hat{T} , then the appropriate matrix element is antisymmetric, so

$$\delta S_J(n_0 \rightarrow n) = -\delta S_J(n \rightarrow n_0). \quad (62)$$

In a general case, when the interaction $\delta\hat{U}$ includes both T-invariant and T-noninvariant components, the S-matrix correction is a sum of symmetric and antisymmetric matrixes.

Taking into account the definition (4), we present the matrix element of the operator $\delta\hat{U}$ in the form

$$\begin{aligned} < JM; n | \delta\hat{U} | JM; n' > = \sum_{\nu\mu\nu'\mu'} C_{j\nu I\mu}^{JM} C_{j'\nu' I'\mu'}^{JM} \times \\ &\times < j\nu; l | \left(\int d\tau \psi_\alpha^*(\tau) \delta\hat{U} \psi_{\alpha'}(\tau) \right) | j'\nu'; l' >. \end{aligned} \quad (63)$$

Let us consider in greater detail a simplest variant, when integrals over internal variables τ are single-particle operators. If the interaction $\delta\hat{U}$ violates P-invariance only, then

$$\int d\tau \psi_\alpha^*(\tau) \delta\hat{U}_P \psi_{\alpha'}(\tau) = \delta_{II'} \delta_{\mu\mu'} \delta_{\pi\pi'} \frac{R}{2\hbar} \left(U_P^{(ii')}(r) \hat{\sigma} \hat{\mathbf{p}} + \hat{\sigma} \hat{\mathbf{p}} U_P^{(ii')}(r) \right), \quad (64)$$

where $\hat{\sigma} = 2\hat{s}$, \hat{s} is an operator of a neutron spin, and $\hat{\mathbf{p}} = -i\hbar\partial/\partial\mathbf{r}$ is a momentum operator. Function $U_P^{(ii')}(r)$ has dimensions of energy owing to a factor R/\hbar . Similarly, a single-particle operator, violating P- and T-invariance, takes the form

$$\int d\tau \psi_{\alpha}^*(\tau) \delta \hat{U}_{PT} \psi_{\alpha'}(\tau) = \delta_{II'} \delta_{\mu\mu'} \delta_{\pi\pi'} \frac{1}{2R} \left(U_{PT}^{(ii')}(r) \hat{\sigma} \mathbf{r} + \hat{\sigma} \mathbf{r} U_{PT}^{(ii')}(r) \right). \quad (65)$$

Functions $U_P^{(ii')}(r)$ and $U_{PT}^{(ii')}(r)$ are real and symmetric.

Matrix element of the single-particle P-noninvariant operator (64) is given by formula

$$\langle j\nu; l | \hat{\sigma} \hat{\mathbf{p}} | j'\nu'; l' \rangle f(r) = \delta_{jj'} \delta_{\nu\nu'} \sqrt{3} i^{l-l'} U(jl' \frac{1}{2} 1, \frac{1}{2} l) (l \| f(r) \| l'), \quad (66)$$

where $U(abcd, ef) = ((2e+1)(2f+1))^{1/2} W(abcd, ef)$ is a normalized Racah function [45], and $(l \| f(r) \| l')$ is a reduced matrix element. We have for s- and p-waves

$$\begin{aligned} (0 \| f(r) \| 0) &= (1 \| f(r) \| 1) = 0, \\ (0 \| f(r) \| 1) &= i\hbar \left(\frac{df(r)}{dr} + 2 \frac{f(r)}{r} \right), \quad (1 \| f(r) \| 0) = -\frac{i\hbar}{\sqrt{3}} \frac{df(r)}{dr}. \end{aligned} \quad (67)$$

Similarly for P-, T-noninvariant interaction we obtain

$$\langle j\nu; l | \hat{\sigma} \mathbf{r} | j'\nu'; l' \rangle = \delta_{jj'} \delta_{\nu\nu'} \sqrt{3} i^{l-l'} U(jl' \frac{1}{2} 1, \frac{1}{2} l) \left(\frac{2l'+1}{2l+1} \right)^{1/2} C_{10\nu'0}^{l0} r. \quad (68)$$

We have already assumed, that the weak interaction, $\delta \hat{U}$, mixes only the states of a target with the same spins, I , projections, μ , and parities, π (see Eqs.(64) and (65)). We consider now a case, when the strong interaction \hat{U} is also diagonal on spins, I , projections, μ , and parities, π , of the inner target states, as well as on orbital, l , and total, j , neutron angular momenta. This is indeed the case in the model of neutron-nucleus interaction studied below. So we write the radial functions in the form

$$\frac{F_{non}^J(r)}{r} = \delta_{I_0 I} \delta_{\mu_0 \mu} \delta_{\pi_0 \pi} \delta_{l_0 l} \delta_{j_0 j} \psi_{lj}^{J(i)}(r). \quad (69)$$

Corrections to S-matrix, describing elastic transitions from s- to p-wave and vice versa, caused by the single-particle P-noninvariant interaction (64), have then the form

$$\begin{aligned} S_J^P(0 \frac{1}{2} \rightarrow 1 \frac{1}{2}) &= S_J^P(1 \frac{1}{2} \rightarrow 0 \frac{1}{2}) = ik_0 R \frac{2m}{\hbar^2} \sum_{ii'} \int_0^\infty r^2 dr \left(\psi_{0\frac{1}{2}}^{J(i)} U_P^{(ii')} \frac{d\psi_{1\frac{1}{2}}^{J(i')}}{dr} - \right. \\ &\quad \left. - \frac{\psi_{0\frac{1}{2}}^{J(i)}}{dr} U_P^{(ii')} \psi_{1\frac{1}{2}}^{J(i')} + 2\psi_{0\frac{1}{2}}^{J(i)} \frac{U_P^{(ii')}}{r} \psi_{1\frac{1}{2}}^{J(i')} \right). \end{aligned} \quad (70)$$

Similarly for corrections, which are due to the P-,T-invariant interaction (65), we find

$$S_J^{PT}(0\frac{1}{2} \rightarrow 1\frac{1}{2}) = -S_J^{PT}(1\frac{1}{2} \rightarrow 0\frac{1}{2}) = k_0 R \frac{2m}{\hbar^2} \sum_{ii'} \int_0^\infty r^2 dr \psi_{0\frac{1}{2}}^{J(i)} \frac{2r U_{PT}^{(ii')}}{R^2} \psi_{1\frac{1}{2}}^{J(i')}. \quad (71)$$

Thus, calculating in the framework of the model the radial functions $\psi_{lj}^{J(i)}(r)$, we get the asymmetries of a total cross section and the angles of spin rotation (31)-(34).

The simplest model is, certainly, the model of potential neutron-nucleus interaction. Earlier P-noninvariant effects, caused by single-particle potential of the type (64), have been studied in Refs.[46, 14]. In Ref.[46] a case of a spherical square well was investigated, while in Ref.[14] an optical potential of a Woods-Saxon type was considered. To account for the elastic channel only one should remove in the formulas (70), (71) the sums over excited states i and i' of a target. Thus substituting the S-matrix elements, $S_J^P(0\frac{1}{2} \rightarrow 1\frac{1}{2}) = S_J^P(1\frac{1}{2} \rightarrow 0\frac{1}{2})$, to Eq.(31), we reproduce the expression for P-noninvariant asymmetry of a total cross section, used in Ref.[14].

5 Model for neutron resonance

5.1 Exact expressions

Let us consider a simple model for a resonance, resulted from a coupling of an incident neutron with one of excited levels of a target. In an elastic channel a neutron interacts with a nucleus, being in a ground state with a spin I , parity π and energy ε_0 . An energy $\varepsilon = E - \varepsilon_0$ of a relative motion is expressed by a momentum of an incident neutron, $\hbar k_0$, in the ordinary way $\varepsilon = (\hbar k_0)^2/2m$ (see Eq.(11)). We assume for simplicity, as was already stated in the previous section, that in the process of target excitation spin, I , projection, μ , and parity, π , do not change (see Eq.(69)). It means, that the matrix elements of the operator of P- and T-invariant strong interaction, \hat{U} , entering in Eq.(39), is diagonal on I , π and, therefore, j and l

$$\langle JM; n | \hat{U} | JM; n' \rangle = \delta_{II'} \delta_{\pi\pi'} \delta_{jj'} \delta_{ll'} U_{ljJ}^{(ii')}(r). \quad (72)$$

Also used is an independence of the matrix element on the quantum number M due to R-invariance of the interaction \hat{U} . Thus, orbital, l , and total, j , neutron angular momenta do not change in the model during the interaction with a nucleus (recall, we are dealing now only with the strong interaction). Potentials $U_{ljJ}^{(ii')}(r)$ are spherically symmetric, as they result from an integration over all internal variables of a target and spin-angular variables of an incident neutron. According to Eq.(42) these potentials are real and symmetric on indexes i and i' .

We assume also, that there exists only one inelastic channel in each partial wave with quantum numbers l, j and J , related with an excitation of a target level with an energy ε_{1ljJ} . Thus we obtain in each partial wave a scattering problem with two coupled channels — elastic and inelastic. For simplicity we shall use only an index l to label partial wave, omitting j and J . Let $\varepsilon_l = \varepsilon_{1ljJ} - \varepsilon_0$ is an energy of target excitation in the partial wave l .

We introduce simplified designations for radial functions: $F_l^{(0)}(r) = F_{n_0 n_0}^J(r)$ in the elastic channel and $F_l^{(1)}(r) = F_{n_0 n_1}^J(r)$ in the inelastic one. According to Eqs.(39) and (41) these functions satisfy the equations

$$\frac{d^2 F_l^{(n)}}{dr^2} - \frac{l(l+1)}{r^2} F_l^{(n)} - \sum_{n'} \frac{2m U_l^{(nn')}(r)}{\hbar^2} F_l^{(n')} + k_{nl}^2 F_l^{(n)} = 0, \quad (73)$$

where $n, n' = 0, 1$. We have for wave numbers k_0 and k_{1l} in the elastic and inelastic channels (see Eqs.(11) and (12))

$$k_0 = \frac{1}{R} \left(\frac{\varepsilon}{\omega} \right)^{1/2}, \quad k_{1l} = \frac{1}{R} \left(\frac{\varepsilon - \varepsilon_l}{\omega} \right)^{1/2}. \quad (74)$$

A quantity

$$\omega = \frac{\hbar^2}{2mR^2} \quad (75)$$

is on the scale of one-particle excitation energy for potential of characteristic radius R . We rewrite the boundary conditions (18) for the radial functions in the form

$$F_l^{(n)}(r)|_{r>R} = \frac{1}{2(k_0 k_{nl})^{1/2}} \left(\delta_{n0} k_{nl} r h_l^{(-)}(k_{nl} r) + S^{(n)}(l \rightarrow l) k_{nl} r h_l^{(+)}(k_{nl} r) \right), \quad (76)$$

where the S-matrix elements $S^{(0)}(l \rightarrow l)$ describe elastic scattering, while $S^{(1)}(l \rightarrow l)$ correspond to transitions from the elastic channel to the inelastic one.

We take all the potentials to be square wells of the same radius R

$$U_l^{(nn)}(r) = \begin{cases} 0, & r > R; \\ -U_l^{(n)}, & r < R; \end{cases} \quad (77)$$

$$U_l^{(01)}(r) = U_l^{(10)}(r) = \begin{cases} 0, & r > R; \\ -W_l, & r < R. \end{cases} \quad (78)$$

Then the equations (73) can be solved analytically. Regular solutions in the region $r < R$ are of the form

$$\begin{cases} F_l^{(0)}(r) = \left(\frac{R}{k_0} \right)^{1/2} \left(A_l \mathfrak{x}_l r j_l(\mathfrak{x}_l r) + A'_l \mathfrak{x}'_l r j_l(\mathfrak{x}'_l r) \right), \\ F_l^{(1)}(r) = \left(\frac{R}{k_0} \right)^{1/2} \left(B_l \mathfrak{x}_l r j_l(\mathfrak{x}_l r) + B'_l \mathfrak{x}'_l r j_l(\mathfrak{x}'_l r) \right). \end{cases} \quad (79)$$

Recall that the spherical Bessel functions, $j_l(x)$, satisfy the equation

$$\frac{d^2}{dx^2}(xj_l(x)) - \left(\frac{l(l+1)}{x^2} - 1\right)xj_l(x) = 0. \quad (80)$$

A factor $(R/k_0)^{1/2}$ is separated in the formulas (79) to simplify final expressions.

Substituting solutions (79) in Eqs.(73) and equating factors at Bessel functions of the same argument, we get

$$\begin{cases} -A_l \mathfrak{x}_l^2 + A_l \frac{2m(U_l^{(0)} + \varepsilon)}{\hbar^2} + B_l \frac{2mW_l}{\hbar^2} = 0, \\ -B_l \mathfrak{x}_l^2 + B_l \frac{2m(U_l^{(1)} + \varepsilon - \varepsilon_l)}{\hbar^2} + A_l \frac{2mW_l}{\hbar^2} = 0, \end{cases} \quad (81)$$

and precisely the same set of equations for A'_l and B'_l . The set (81) is tractable if its determinant equals zero. This gives the quadratic equation for a quantity \mathfrak{x}_l^2 , two roots of which are \mathfrak{x}_l^2 and $\mathfrak{x}_l'^2$. We find for \mathfrak{x}_l and \mathfrak{x}_l'

$$\mathfrak{x}_l = \left(\frac{2m}{\hbar^2}(U_l^{(0)} + \varepsilon + \Delta_l)\right)^{1/2} = \frac{1}{R} \left(\frac{U_l^{(0)} + \varepsilon + \Delta_l}{\omega}\right)^{1/2}, \quad (82)$$

$$\mathfrak{x}_l' = \left(\frac{2m}{\hbar^2}(U_l^{(1)} + \varepsilon - \varepsilon_l - \Delta_l)\right)^{1/2} = \frac{1}{R} \left(\frac{U_l^{(1)} + \varepsilon - \varepsilon_l - \Delta_l}{\omega}\right)^{1/2}, \quad (83)$$

where

$$\Delta_l = \left(\left(\frac{U_l^{(0)} - U_l^{(1)} + \varepsilon_l}{2}\right)^2 + W_l^2\right)^{1/2} - \left(\frac{U_l^{(0)} - U_l^{(1)} + \varepsilon_l}{2}\right). \quad (84)$$

We assume that

$$U_l^{(0)} - U_l^{(1)} + \varepsilon_l > 0, \quad (85)$$

therefore in the absence of channel coupling $W_l \rightarrow 0$ we have $\Delta_l \rightarrow 0$, so \mathfrak{x}_l and \mathfrak{x}_l' become equal to the wave numbers in the elastic and inelastic channels, respectively, in the region $r < R$. Factors B_l and B'_l take the form

$$B_l = \lambda_l A_l, \quad B'_l = -\frac{1}{\lambda_l} A'_l, \quad (86)$$

where

$$\lambda_l = \frac{\Delta_l}{W_l}. \quad (87)$$

Functions (76) and (79), and their first derivatives should be joined at the point $r = R$. This gives four equations for four unknown quantities A_l , A'_l , $S^{(0)}(l \rightarrow l)$ and $S^{(1)}(l \rightarrow l)$. As far as a relation

$$\frac{d}{dx}(x^{l+1}f_l(x)) = x^{l+1}f_{l-1}(x), \quad (88)$$

is valid for spherical Bessel and Hankel functions, it is convenient to join the first derivatives of the radial functions, multiplied by r^l .

We get for joining conditions in the inelastic channel

$$\begin{cases} B_l \mathfrak{x}_l R j_l(\mathfrak{x}_l R) + B'_l \mathfrak{x}'_l R j_l(\mathfrak{x}'_l R) = \frac{1}{2(k_{1l}R)^{1/2}} S^{(1)}(l \rightarrow l) k_{1l} R h_l^{(+)}(k_{1l} R), \\ B_l(\mathfrak{x}_l R)^2 j_{l-1}(\mathfrak{x}_l R) + B'_l(\mathfrak{x}'_l R)^2 j_{l-1}(\mathfrak{x}'_l R) = \\ = \frac{1}{2(k_{1l}R)^{1/2}} S^{(1)}(l \rightarrow l) (k_{1l} R)^2 h_{l-1}^{(+)}(k_{1l} R). \end{cases} \quad (89)$$

Consider a case, when the inelastic channel is closed, that is, $\varepsilon < \varepsilon_l$, and the channel coupling is absent, so $B_l = 0$. Then Eqs.(89) specify an energy $\varepsilon - \varepsilon_l < 0$ of a bound state of a neutron in the square potential of radius R and depth $U_l^{(1)}$. This energy can be found by equating a determinant

$$D'_l(\varepsilon) = \mathfrak{x}'_l R j_l(\mathfrak{x}'_l R) (k_{1l} R)^2 h_{l-1}^{(+)}(k_{1l} R) - (\mathfrak{x}'_l R)^2 j_{l-1}(\mathfrak{x}'_l R) k_{1l} R h_l^{(+)}(k_{1l} R). \quad (90)$$

to zero. We introduce by analogy a function

$$D_l(\varepsilon) = \mathfrak{x}_l R j_l(\mathfrak{x}_l R) (k_{1l} R)^2 h_{l-1}^{(+)}(k_{1l} R) - (\mathfrak{x}_l R)^2 j_{l-1}(\mathfrak{x}_l R) k_{1l} R h_l^{(+)}(k_{1l} R), \quad (91)$$

and return to the analysis of a general case. Thus eliminating $S^{(1)}(l \rightarrow l)$ from Eqs.(89), we get a relation between the factors B_l and B'_l

$$B_l D_l(\varepsilon) + B'_l D'_l(\varepsilon) = 0. \quad (92)$$

We turn now to joining conditions in the elastic channel

$$\begin{cases} A_l \mathfrak{x}_l R j_l(\mathfrak{x}_l R) + A'_l \mathfrak{x}'_l R j_l(\mathfrak{x}'_l R) = \\ = \frac{1}{2(k_0 R)^{1/2}} (k_0 R h_l^{(-)}(k_0 R) + S^{(0)}(l \rightarrow l) k_0 R h_l^{(+)}(k_0 R)), \\ A_l(\mathfrak{x}_l R)^2 j_{l-1}(\mathfrak{x}_l R) + A'_l(\mathfrak{x}'_l R)^2 j_{l-1}(\mathfrak{x}'_l R) = \\ = \frac{1}{2(k_0 R)^{1/2}} ((k_0 R)^2 h_{l-1}^{(-)}(k_0 R) + S^{(0)}(l \rightarrow l) (k_0 R)^2 h_{l-1}^{(+)}(k_0 R)). \end{cases} \quad (93)$$

Eliminating $S^{(0)}(l \rightarrow l)$ and taking into account, that

$$h_l^{(-)}(x)h_{l-1}^{(+)}(x) - h_{l-1}^{(-)}(x)h_l^{(+)}(x) = \frac{2i}{x^2}, \quad (94)$$

we obtain a relation between the factors A_l and A'_l

$$\begin{aligned} & A_l \left(\mathfrak{x}_l R j_l(\mathfrak{x}_l R) (k_0 R)^2 h_{l-1}^{(+)}(k_0 R) - (\mathfrak{x}_l R)^2 j_{l-1}(\mathfrak{x}_l R) k_0 R h_l^{(+)}(k_0 R) \right) + \\ & + A'_l \left(\mathfrak{x}'_l R j_l(\mathfrak{x}'_l R) (k_0 R)^2 h_{l-1}^{(+)}(k_0 R) - (\mathfrak{x}'_l R)^2 j_{l-1}(\mathfrak{x}'_l R) k_0 R h_l^{(+)}(k_0 R) \right) = \\ & = i(k_0 R)^{1/2}. \end{aligned} \quad (95)$$

This equation along with (86) and (92) enables us to find the factors A_l and A'_l (and, certainly, B_l and B'_l)

$$A_l = i(k_0 R)^{1/2} \frac{D'_l(\varepsilon)}{Z_l(\varepsilon)}, \quad A'_l = i(k_0 R)^{1/2} \frac{\lambda_l^2 D_l(\varepsilon)}{Z_l(\varepsilon)}, \quad (96)$$

where

$$\begin{aligned} Z_l(\varepsilon) = & (k_0 R)^2 h_{l-1}^{(+)}(k_0 R) \left(D'_l(\varepsilon) \mathfrak{x}_l R j_l(\mathfrak{x}_l R) + \lambda_l^2 D_l(\varepsilon) \mathfrak{x}'_l R j_l(\mathfrak{x}'_l R) \right) - \\ & - k_0 R h_l^{(+)}(k_0 R) \left(D'_l(\varepsilon) (\mathfrak{x}_l R)^2 j_{l-1}(\mathfrak{x}_l R) + \lambda_l^2 D_l(\varepsilon) (\mathfrak{x}'_l R)^2 j_{l-1}(\mathfrak{x}'_l R) \right). \end{aligned} \quad (97)$$

Formulas (86) and (96) specify the energy dependent factors A_l , A'_l , B_l and B'_l , and, therefore, radial wave functions (79).

We find now a position of a resonance in the partial wave l . A cross section for elastic scattering is determined by an element of S-matrix, $S^{(0)}(l \rightarrow l)$, or a phase shift, δ_l . These quantities express in terms of a logarithmic derivative of the elastic-channel radial function in the ordinary way [34]

$$S^{(0)}(l \rightarrow l) = \exp(2i\delta_l) = \exp(2i\delta_l^c) \frac{\Phi_l(\varepsilon) + is_l}{\Phi_l(\varepsilon) - is_l}, \quad (98)$$

where

$$\Phi_l(\varepsilon) = \tilde{\Phi}_l(\varepsilon) - d_l, \quad \tilde{\Phi}_l(\varepsilon) = R \frac{dF_l^{(0)}/dr}{F_l^{(0)}} \Big|_R. \quad (99)$$

This results from the joining conditions (93). Phase shifts, δ_l^c , for scattering on an opaque sphere of radius R are given by equation

$$\exp(2i\delta_l^c) = -\frac{h_l^{(-)}(k_0 R)}{h_l^{(+)}(k_0 R)}, \quad (100)$$

while factors of shift, $d_l(\varepsilon)$, and penetrability, $s_l(\varepsilon)$, are real and imaginary parts of expression

$$d_l + is_l = \frac{(xh_l^{(+)}(x))'}{h_l^{(+)}(x)} \Big|_{x=k_0R} = -l + \frac{(k_0R)h_{l-1}^{(+)}(k_0R)}{h_l^{(+)}(k_0R)}. \quad (101)$$

At low energies ($k_0R \ll 1$) the following asymptotic formulas are

$$\delta_l^c \simeq -\frac{(k_0R)^{2l+1}}{(2l-1)!!(2l+1)!!}, \quad d_l \simeq -l, \quad s_l \simeq \frac{(k_0R)^{2l+1}}{((2l-1)!!)^2}. \quad (102)$$

A resonance occurs at an energy E_l , such that $\Phi_l(E_l) = 0$. If we restrict to the first term of the Taylor expansion

$$\Phi_l(\varepsilon) = -\frac{1}{\gamma_l}(\varepsilon - E_l), \quad (103)$$

we find the S-matrix element in the Breit-Wigner form

$$S^{(0)}(l \rightarrow l) = \exp(2i\delta_l^c) \left(1 - \frac{i\Gamma_l}{\varepsilon - E_l + i\frac{\Gamma_l}{2}}\right). \quad (104)$$

A width of a resonance is specified by expression

$$\Gamma_l = 2s_l\gamma_l, \quad (105)$$

where γ_l is a reduced width.

In the model that we consider the logarithmic derivative of the elastic-channel function $F_l^{(0)}(r)$ at the point $r = R$ takes the form

$$\Phi_l(\varepsilon) = \frac{A_l(\varkappa_l R)^2 j_{l-1}(\varkappa_l R) + A'_l(\varkappa'_l R)^2 j_{l-1}(\varkappa'_l R)}{A_l \varkappa_l R j_l(\varkappa_l R) + A'_l \varkappa'_l R j_l(\varkappa'_l R)} - l - d_l. \quad (106)$$

Substituting to this formula the energy dependent factors A_l and A'_l , we obtain the explicit expression for the logarithmic derivative $\Phi_l(\varepsilon)$ and, therefore, for the phase shift δ_l and the partial cross section for elastic scattering.

5.2 Approximate expressions

In this section we study in greater details an energy dependence of the factors A_l , A'_l , B_l and B'_l near a narrow resonance. Such resonance occurs, if a coupling between channels is weak, that is, if $|W_l| \ll \varepsilon_l$. Provided $U_l^{(0)} \sim U_l^{(1)}$, we have $\Delta_l \simeq W_l^2/\varepsilon_l$, so the dimensionless parameter λ_l (87) is small $|\lambda_l| \sim |W_l/\varepsilon_l| \ll 1$. We assume that the energy of incident neutrons is low ($k_0R \ll 1$), so we use the asymptotic formulas (102).

Returning to the exact expression (106) for the logarithmic derivative of the elastic-channel wave function, we notice, that $A_l' = 0$ in the absence of channel coupling. Then Eq.(106) specifies the logarithmic derivative of the radial function for the elastic scattering of a neutron on a spherical potential well of radius R and depth $U_l^{(0)}$

$$\Phi_l^0(\varepsilon) = \frac{\varkappa_l R j_{l-1}(\varkappa_l R)}{j_l(\varkappa_l R)}. \quad (107)$$

Let us show, that the exact expression (106) for $\Phi_l(\varepsilon)$ reduces to $\Phi_l^0(\varepsilon)$ (107) everywhere, except in a small vicinity of a resonance.

Taking into account Eqs.(96), we rewrite the expression (106) in the form

$$\Phi_l(\varepsilon) = \frac{D_l'(\varepsilon)(\varkappa_l R)^2 j_{l-1}(\varkappa_l R) + \lambda_l^2 D_l(\varepsilon)(\varkappa_l' R)^2 j_{l-1}(\varkappa_l' R)}{D_l'(\varepsilon) \varkappa_l R j_l(\varkappa_l R) + \lambda_l^2 D_l(\varepsilon) \varkappa_l' R j_l(\varkappa_l' R)}. \quad (108)$$

We assume that there are no peculiarities in the potential scattering (i.e., in the quantity (107)) in the considered region of low energy. Then, obviously, at $\lambda_l^2 \ll 1$ function $\Phi_l(\varepsilon)$ coincides with the accuracy of λ_l^2 with $\Phi_l^0(\varepsilon)$ everywhere, except in a small vicinity of an energy E_l' , such that $D_l'(E_l') = 0$. We note, that at $\lambda_l = 0$ there exists a bound state of a neutron with an energy $E_l' - \varepsilon_l$ in the inelastic channel (see text before Eq.(90)). So, clearly, the scattering cross section deviates from the potential behavior and, as we shall see, has the resonant nature just in the vicinity of the energy E_l' .

We find now an energy E_l of a resonance. According to equations (74), (82), (83) and definitions (90), (91) all functions, entering into numerator and denominator of the fraction (108), are slow functions of energy. Namely, they change significantly on the scale of ω (75). Therefore the quantity $\Phi_l(\varepsilon)$ (108) is a slow function of energy everywhere, except in a small vicinity of the energy E_l' . In this vicinity we present the function $D_l'(\varepsilon)$ in the form

$$D_l'(\varepsilon) = (\varepsilon - E_l') \frac{dD_l'(\varepsilon)}{d\varepsilon} \Big|_{E_l'}, \quad (109)$$

that is valid, incidentally, on an interval $|\varepsilon - E_l'| \ll \omega$. The derivative, $dD_l'(\varepsilon)/d\varepsilon$, is on the order of $\sim D_l'(\varepsilon)/\omega$. It is easy to show, that the numerator of the fraction (108) becomes zero at the energy

$$E_l = E_l' - \lambda_l^2 \frac{1}{\left(\frac{dD_l'(\varepsilon)}{d\varepsilon}\right)_{E_l'}} \left(\frac{D_l(\varepsilon)(\varkappa_l' R)^2 j_{l-1}(\varkappa_l' R)}{(\varkappa_l R)^2 j_{l-1}(\varkappa_l R)} \right)_{E_l}, \quad (110)$$

and the difference $|E_l - E_l'|$ is small in comparison with ω owing to $\lambda_l^2 \ll 1$. Therefore $dD_l'(\varepsilon)/d\varepsilon|_{E_l'} \simeq dD_l'(\varepsilon)/d\varepsilon|_{E_l}$. The energy E_l (110) is the required energy of a resonance in the partial wave l , occurring due to a coupling between the elastic and inelastic channels.

We study now a behavior of the logarithmic derivative near the energy E_l . Substituting the expansion (109) to Eq.(108) and taking into account the definition (107) and result (110), we obtain

$$\Phi_l(\varepsilon) = \frac{\varepsilon - E_l}{\frac{\varepsilon - E_l}{\Phi_l^0} - \gamma_l}, \quad (111)$$

where $\Phi_l^0 = \Phi_l^0(E_l)$ and

$$\gamma_l = \lambda_l^2 \omega \left(\frac{2\omega D_l^2(\varepsilon)(\varkappa'_l R)^2}{(U_l^{(1)} - \Delta_l) ((\varkappa_l R)^2 j_{l-1}(\varkappa_l R))^2 (k_{1l} R)^2 h_{l-1}^{(+)}(k_{1l} R) h_{l+1}^{(+)}(k_{1l} R)} \right)_{E_l}. \quad (112)$$

We see, that close to E_l the energy dependence of the function $\Phi_l(\varepsilon)$ is determined by the quantity $\gamma_l \sim \lambda_l^2 \omega \ll \omega$. In the small vicinity of E_l

$$|\varepsilon - E_l| \ll \gamma_l |\Phi_l^0| \quad (113)$$

we get the Breit-Wigner formulas (103)-(105). We note, however, that the parametrization (111) holds for much more wide energy interval $|\varepsilon - E_l| \ll \omega$. Thus in the region, lying beyond the reduced width of a resonance,

$$\gamma_l |\Phi_l^0| \ll |\varepsilon - E_l| \ll \omega, \quad (114)$$

the logarithmic derivative $\Phi_l(\varepsilon)$ (111) reduces simply to the potential value (107), taken at the point E_l .

Similarly, substituting the expansion (109) in Eqs.(96), (97) and taking into account Eqs.(86) and (112), we obtain the following expressions for the factors A_l , A'_l , B_l and B'_l on the interval $|\varepsilon - E_l| \ll \omega$

$$A_l(\varepsilon) = \exp(i\delta_l^c)(s_l)^{1/2} \frac{(\varepsilon - E_l - \gamma_l x_l)}{[l]} \left(\frac{1}{(\varkappa_l R)^2 j_{l-1}(\varkappa_l R)} \right)_{E_l}, \quad (115)$$

$$A'_l(\varepsilon) = \exp(i\delta_l^c)(s_l)^{1/2} \frac{\gamma_l x_l}{[l]} \left(\frac{1}{(\varkappa'_l R)^2 j_{l-1}(\varkappa'_l R)} \right)_{E_l}, \quad (116)$$

$$B_l(\varepsilon) = y_l \left(\frac{\gamma_l}{\omega} \right)^{1/2} A_l(\varepsilon), \quad B'_l(\varepsilon) = -\frac{1}{y_l} \left(\frac{\omega}{\gamma_l} \right)^{1/2} A'_l(\varepsilon). \quad (117)$$

Here the designations are introduced for quantities

$$x_l = \left(\frac{(\varkappa_l R)^2 j_{l-1}(\varkappa_l R) (k_{1l} R)^2 h_{l-1}^{(+)}(k_{1l} R)}{D_l(\varepsilon)} \right)_{E_l}, \quad (118)$$

$$y_l = \left(\frac{x_l(U_l^{(1)} - \Delta_l)(\mathfrak{x}_l R)^2 j_{l-1}(\mathfrak{x}_l R) h_{l+1}^{(+)}(k_{1l} R)}{2\omega D_l(\varepsilon)(\mathfrak{x}_l' R)^2} \right)_{E_l}^{1/2}, \quad (119)$$

as well as for a resonance denominator

$$[l] = \varepsilon - E_l + i s_l \left(\gamma_l - \frac{\varepsilon - E_l}{\Phi_l^0} \right). \quad (120)$$

A factor, containing this denominator, has the following asymptotic forms

$$\frac{1}{[l]} \rightarrow \begin{cases} \frac{1}{\varepsilon - E_l + i \frac{\Gamma_l}{2}}, & \text{if } |\varepsilon - E_l| \ll \gamma_l |\Phi_l^0|; \\ \frac{1}{\varepsilon - E_l} \exp\left(i \frac{s_l}{\Phi_l^0}\right), & \text{if } \gamma_l |\Phi_l^0| \ll |\varepsilon - E_l| \ll \omega. \end{cases} \quad (121)$$

Due to this factor all quantities (115)-(117) increase resonantly in the small vicinity of E_l . Comparing the coefficients at this factor, however, it is easy to show, that

$$|B_l| \ll |A_l| \sim |A_l'| \ll |B_l'|, \quad \text{if } |\varepsilon - E_l| \ll \gamma_l |\Phi_l^0|. \quad (122)$$

We consider now the region (114), lying outside the resonance. Clearly, the factors A_l and B_l take the constant values, while A_l' and B_l' fall off with a spacing from the resonance as follows $\sim 1/(\varepsilon - E_l)$. Therefore in this region

$$|A_l'| \ll |A_l|, \quad \text{if } \gamma_l |\Phi_l^0| \ll |\varepsilon - E_l| \ll \omega. \quad (123)$$

The superiority of B_l' over A_l is still persist in the interval, which goes far beyond the reduced width of the resonance

$$|A_l| \ll |B_l'|, \quad \text{if } |\varepsilon - E_l| \ll \gamma_l^{1/2} \omega^{1/2} \frac{x_l}{y_l} \left| \frac{(\mathfrak{x}_l R)^2 j_{l-1}(\mathfrak{x}_l R)}{(\mathfrak{x}_l' R)^2 j_{l-1}(\mathfrak{x}_l' R)} \right|_{E_l}. \quad (124)$$

On the other hand, the factor B_l' surpasses B_l practically everywhere over the region of applicability of Eqs.(115)-(117)

$$|B_l| \ll |B_l'|, \quad \text{if } |\varepsilon - E_l| \ll \omega \frac{x_l}{y_l^2} \left| \frac{(\mathfrak{x}_l R)^2 j_{l-1}(\mathfrak{x}_l R)}{(\mathfrak{x}_l' R)^2 j_{l-1}(\mathfrak{x}_l' R)} \right|_{E_l}. \quad (125)$$

Thus, far away from the resonance ($|\varepsilon - E_l| > \gamma_l^{1/2} \omega^{1/2}$) the term, proportional to the factor A_l , dominates in the wave function (79). Here, we are dealing with potential elastic scattering. The probability density to find a neutron inside a target is proportional to the factor of penetrability of centrifugal barrier and potential jump at a nuclear boundary [47]

$$(|A_l|^2)_{pot} \sim s_l. \quad (126)$$

While in the resonance ($\varepsilon \sim E_l$) we have for the probability density in the elastic and inelastic channels

$$\left(|A_l|^2\right)_{res} \sim \left(|A'_l|^2\right)_{res} \sim \frac{1}{s_l}, \quad (127)$$

$$\left(|B'_l|^2\right)_{res} \sim \frac{\omega}{s_l \gamma_l}. \quad (128)$$

An increase of probability density in the resonance can be qualitatively explained as follows. The characteristic time of change of a wave packet, made from one-particle functions inside a nucleus, is $T = \hbar/\omega$. An exit of the packet is retarded by centrifugal barrier and potential jump. Therefore the exit time is evaluated by $T_l = T/s_l$, so

$$\left(|A_l|^2\right)_{res} \sim \left(|A'_l|^2\right)_{res} \sim \frac{T_l}{T}. \quad (129)$$

As for the inelastic channel, in the model that we consider an exit of neutrons is retarded by a weak coupling between the channels. Taking into account the definition of the resonance width (105), we can write

$$\left(|B'_l|^2\right)_{res} \sim \frac{\omega}{\Gamma_l} \sim \frac{\tau_l}{T}, \quad (130)$$

where $\tau_l = \hbar/\Gamma_l$ is the usually defined life time of compound nucleus. Thus we see, that both in the elastic and inelastic channels an enhancement in probability density corresponds to an increase of time spent by a neutron inside a target.

In closing of this section we note, that the quantities x_l and y_l are real until the inelastic channel is closed ($\varepsilon < \varepsilon_l$), so the phases of the factors A_l , A'_l , B_l and B'_l coincide. Moreover, these phases are equal to the phase shift for elastic scattering in the partial wave l

$$\begin{aligned} \delta_l &= \delta_l^c + \arg \frac{1}{[l]} = \delta_l^c + \operatorname{arctg} \frac{s_l}{\Phi_l(\varepsilon)} \rightarrow \\ &\rightarrow \begin{cases} \delta_l^c + \operatorname{arctg} \frac{\Gamma_l(\varepsilon)}{2(E_l - \varepsilon)}, & \text{if } |\varepsilon - E_l| \ll \gamma_l |\Phi_l^0|; \\ \delta_l^c + \frac{s_l}{\Phi_l^0}, & \text{if } \gamma_l |\Phi_l^0| \ll |\varepsilon - E_l| \ll \omega. \end{cases} \end{aligned} \quad (131)$$

In a general way, this results from Eqs.(93). Indeed, a factor $\exp(i\delta_l)$ can be separated as a common phase factor on the right sides of these equations.

6 P- and T-noninvariant mixing of s- and p-wave resonances

In the previous section the model for resonance in the arbitrary partial wave l was constructed. It is easy to apply the results obtained to describe a close-lying pair of two narrow s- and p-wave resonances. The needed spherical Bessel and Hankel functions are of the form

$$\begin{aligned} j_{-1}(x) &= \frac{\cos x}{x}, \quad j_0(x) = \frac{\sin x}{x}, \quad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}, \\ h_{-1}^{(+)}(x) &= \frac{e^{ix}}{x}, \quad h_0^{(+)}(x) = \frac{-i}{x}e^{ix}, \quad h_1^{(+)}(x) = -\left(\frac{1}{x} + \frac{i}{x^2}\right)e^{ix}, \\ h_2^{(+)}(x) &= \left(\frac{i}{x} - \frac{3}{x^2} - \frac{3i}{x^3}\right)e^{ix}. \end{aligned} \quad (132)$$

We assume, that a distance between resonances is much less than a one-particle energy ω . So we use the result (125) and neglect the components $\sim B_l$ of the radial functions in the inelastic channel (see Eq.(79)).

The parity violating interaction of the form (64) leads to the elements of S-matrix (70), corresponding to transitions from s- to p-wave. Similarly the interaction of the form (65), violating the space parity and time reversal symmetry, results in the corrections (71) to S-matrix. The substitution of the radial functions (79) to the Eqs.(70), (71) gives the elements of S-matrix, caused by P- and T-violation. Thus, in the model considered the transmission asymmetry ϵ_P (36) and the angle of spin rotation χ_P (32) are obtainable from the formulas

$$\frac{\epsilon_P}{n} = -\frac{4\pi}{k^2}g_J\text{Re}S_J^P(0\frac{1}{2} \rightarrow 1\frac{1}{2}), \quad \frac{\chi_P}{n} = \frac{4\pi}{k^2}g_J\text{Im}S_J^P(0\frac{1}{2} \rightarrow 1\frac{1}{2}). \quad (133)$$

Similarly the quantities ϵ_{PT} (36) and χ_{PT} (34) are determined by the following expressions

$$\frac{\epsilon_{PT}}{n} = -\frac{4\pi}{k^2}C_{J\frac{1}{2}}\text{Im}S_J^{PT}(0\frac{1}{2} \rightarrow 1\frac{1}{2}), \quad \frac{\chi_{PT}}{n} = -\frac{4\pi}{k^2}C_{J\frac{1}{2}}\text{Re}S_J^{PT}(0\frac{1}{2} \rightarrow 1\frac{1}{2}). \quad (134)$$

It is apparent here, that P- and T-noninvariant effects enhance near the resonances. Indeed, it was shown in the previous section that the amplitudes A_l , A'_l and B'_l of radial wave functions (79) peak at resonance energies (see Eqs.(115)-(117)). Thus the radial integrals (70), (71), that is, the corrections to S-matrix, increase too. To get the explicit expressions for these corrections the form of the radial dependence of potentials $U_P^{(ii')}(r)$ and $U_{PT}^{(ii')}(r)$ should be chosen.

By analogy with potentials of strong interaction (see Eqs.(77), (78)), we take P- and P-,T-noninvariant potentials to be spherical square wells of radius R

$$U_{P(PT)}^{(ii')}(r) = \begin{cases} 0, & r > R; \\ U_{P(PT)}^{(ii')}, & r < R. \end{cases} \quad (135)$$

Now it is convenient to rewrite the Eq.(70) in the form

$$S_J^P(0\frac{1}{2} \rightarrow 1\frac{1}{2}) = ik_0 R \frac{2m}{\hbar^2} \times \\ \times \sum_{ii'} \int_0^\infty dr F_s^{(i)} \left\{ 2U_P^{(ii')}(r) \left(\frac{dF_p^{(i')}}{dr} + \frac{F_p^{(i')}}{r} \right) + \frac{dU_P^{(ii')}(r)}{dr} F_p^{(i')} \right\}, \quad (136)$$

as the differentiation of the step function $U_P^{(ii')}(r)$ gives the easily integrated δ -function. At the same time replacing the functions $\psi_l^{(i)}$ by $F_l^{(i)}$ we get for the amplitude of P-,T-noninvariant transition (71)

$$S_J^{PT}(0\frac{1}{2} \rightarrow 1\frac{1}{2}) = k_0 R \frac{2m}{\hbar^2} \sum_{ii'} \int_0^\infty dr F_s^{(i)} \frac{2r U_{PT}^{(ii')}(r)}{R^2} F_p^{(i')}. \quad (137)$$

Substituting the explicit expressions for radial functions in these formulas (see Eq.(79)), we can separate contributions of four types to P- and P-,T-noninvariant corrections to S-matrix

$$S_J^{P(PT)}(0\frac{1}{2} \rightarrow 1\frac{1}{2}) = \delta S_{P(PT)}^{(00)} + \delta S_{P(PT)}^{(01p)} + \delta S_{P(PT)}^{(1s0)} + \delta S_{P(PT)}^{(1s1p)}. \quad (138)$$

The corrections of the first type $\delta S_{P(PT)}^{(00)}$ are related with a mixing of radial functions of the elastic channel, $F_s^{(0)}$ and $F_p^{(0)}$, and correspond to direct transition of a neutron from s- to p-wave due to weak interaction $\delta \hat{U}_{P(PT)}$ with target nucleons. A target nucleus remains in a ground state. The corrections of the second type $\delta S_{P(PT)}^{(01p)}$ are caused by transitions of a neutron from the s-wave elastic channel to the p-wave inelastic one with an excitation of a target to the state with an energy ε_{1p} . Similarly the corrections of the third type $\delta S_{P(PT)}^{(1s0)}$ correspond to neutron transitions from the inelastic s-wave channel to the elastic p-wave one, when a target passes from an excited level with the energy ε_{1s} to a ground state. Finally, the corrections of fourth type $\delta S_{P(PT)}^{(1s1p)}$ correspond to a mixing of s- and p-wave functions of the inelastic channels, $F_s^{(1)}$ and $F_p^{(1)}$, with a transition of a target from an excited level ε_{1s} to another ε_{1p} one.

In the framework of the model considered we have for these P- and P-,T-noninvariant corrections

$$\delta S_{P(PT)}^{(00)} = \xi_{P(PT)} \frac{U_{P(PT)}^{(00)}}{\omega} \left(A_s A_p f_{P(PT)}(\mathfrak{x}_s, \mathfrak{x}_p) + A_s A'_p f_{P(PT)}(\mathfrak{x}_s, \mathfrak{x}'_p) + \right. \\ \left. + A'_s A_p f_{P(PT)}(\mathfrak{x}'_s, \mathfrak{x}_p) + A'_s A'_p f_{P(PT)}(\mathfrak{x}'_s, \mathfrak{x}'_p) \right), \quad (139)$$

$$\delta S_{P(PT)}^{(01p)} = \xi_{P(PT)} \frac{U_{P(PT)}^{(01p)}}{\omega} \left(A_s B'_p f_{P(PT)}(\mathfrak{x}_s, \mathfrak{x}'_p) + A'_s B'_p f_{P(PT)}(\mathfrak{x}'_s, \mathfrak{x}'_p) \right), \quad (140)$$

$$\delta S_{P(PT)}^{(1s0)} = \xi_{P(PT)} \frac{U_{P(PT)}^{(1s0)}}{\omega} \left(B'_s A_p f_{P(PT)}(\mathfrak{x}'_s, \mathfrak{x}_p) + B'_s A'_p f_{P(PT)}(\mathfrak{x}'_s, \mathfrak{x}'_p) \right), \quad (141)$$

$$\delta S_{P(PT)}^{(1s1p)} = \xi_{P(PT)} \frac{U_{P(PT)}^{(1s1p)}}{\omega} B'_s B'_p f_{P(PT)}(\mathfrak{x}'_s, \mathfrak{x}'_p), \quad (142)$$

where $\xi_P = i$, $\xi_{PT} = 1$ and the following designations for integrals are used

$$f_P(\mathfrak{x}_s, \mathfrak{x}_p) = 2 \int_0^R dr \sin \mathfrak{x}_s r \mathfrak{x}_p \sin \mathfrak{x}_p r - \sin \mathfrak{x}_s R \left(\frac{\sin \mathfrak{x}_p R}{\mathfrak{x}_p R} - \cos \mathfrak{x}_p R \right), \quad (143)$$

$$f_{PT}(\mathfrak{x}_s, \mathfrak{x}_p) = \frac{2}{R^2} \int_0^R r dr \sin \mathfrak{x}_s r \left(\frac{\sin \mathfrak{x}_p r}{\mathfrak{x}_p r} - \cos \mathfrak{x}_p r \right). \quad (144)$$

These integrals are easily calculated

$$\begin{aligned} f_P(\mathfrak{x}_s, \mathfrak{x}_p) &= \mathfrak{x}_p R \left(\frac{\sin(\mathfrak{x}_s - \mathfrak{x}_p)R}{(\mathfrak{x}_s - \mathfrak{x}_p)R} - \frac{\sin(\mathfrak{x}_s + \mathfrak{x}_p)R}{(\mathfrak{x}_s + \mathfrak{x}_p)R} \right) - \\ &\quad - \sin \mathfrak{x}_s R \left(\frac{\sin \mathfrak{x}_p R}{\mathfrak{x}_p R} - \cos \mathfrak{x}_p R \right), \end{aligned} \quad (145)$$

$$\begin{aligned} f_{PT}(\mathfrak{x}_s, \mathfrak{x}_p) &= \frac{1}{\mathfrak{x}_p R} \left(\frac{\sin(\mathfrak{x}_s - \mathfrak{x}_p)R}{(\mathfrak{x}_s - \mathfrak{x}_p)R} - \frac{\sin(\mathfrak{x}_s + \mathfrak{x}_p)R}{(\mathfrak{x}_s + \mathfrak{x}_p)R} \right) - \\ &\quad - \frac{1}{(\mathfrak{x}_s - \mathfrak{x}_p)R} \left(\frac{\sin(\mathfrak{x}_s - \mathfrak{x}_p)R}{(\mathfrak{x}_s - \mathfrak{x}_p)R} - \cos(\mathfrak{x}_s - \mathfrak{x}_p)R \right) - \\ &\quad - \frac{1}{(\mathfrak{x}_s + \mathfrak{x}_p)R} \left(\frac{\sin(\mathfrak{x}_s + \mathfrak{x}_p)R}{(\mathfrak{x}_s + \mathfrak{x}_p)R} - \cos(\mathfrak{x}_s + \mathfrak{x}_p)R \right). \end{aligned} \quad (146)$$

The depths of P- and T-noninvariant potentials $U_{P(PT)}^{(ii')}$ are external parameters of the model. However to compare the contributions (139)-(142) to the observables we need some estimates for these depths. We assume, that P- or P-,T-noninvariant operator of neutron-nucleons interaction $\delta \hat{U}_{P(PT)}$ is a sum of pairwise operators. Then the functions $U_{P(PT)}^{(ii')}(r)$ on the right parts of Eqs.(64)-(65) are single-particle matrix elements. In the case of parity nonconserving interaction a diagonal single-particle matrix element is usually evaluated by

$$U_P^{(00)} \sim \langle \hat{u}_P \rangle_{s.p.} \sim G m_\pi^2 \Omega \sim 0.1 \text{ eV}, \quad (147)$$

where $G = 10^{-5}/m_p^2$ is a Fermi constant, m_π is a pion mass, and $\Omega \sim 1 \text{ MeV}$ is a characteristic one-particle energy. In the case of P-,T-noninvariant interaction we assume, that

$$U_{PT}^{(00)} \sim \langle \hat{u}_{PT} \rangle_{s.p.} \sim \phi G m_\pi^2 \Omega, \quad (148)$$

where $\phi < 10^{-3}$ according the present data.

To estimate nondiagonal single-particle matrix elements we use a well known procedure [19]-[21]. We present a target wave function, which describes a highly excited state, as a superposition of many simple configurations

$$\psi_\alpha = \sum_{k=1}^N c_k \psi_k. \quad (149)$$

Here $|c_k| \sim 1/N^{1/2}$ owing to normalization condition. A number N is usually evaluated as $N \sim \omega/D$, where D is a mean distance between excited levels. In this sum few configurations differ from a ground state ψ_0 by an excitation of one particle only. Therefore

$$U_{P(PT)}^{(0\ 1p)} \sim U_{P(PT)}^{(1s\ 0)} \sim \alpha |\hat{u}_{P(PT)}|0\rangle \sim \frac{1}{N^{1/2}} \langle \hat{u}_{P(PT)} \rangle_{s.p.} \sim \frac{U_{P(PT)}^{(00)}}{N^{1/2}}. \quad (150)$$

Similarly we obtain

$$U_{P(PT)}^{(1s\ 1p)} \sim \alpha |\hat{u}_{P(PT)}|\alpha'\rangle \sim \frac{1}{N} N^{1/2} \langle \hat{u}_{P(PT)} \rangle_{s.p.} \sim \frac{U_{P(PT)}^{(00)}}{N^{1/2}}, \quad (151)$$

where, as usual, a noncoherent sum of $\sim N$ single-particle matrix elements between simple configurations is estimated by $N^{1/2} \langle \hat{u} \rangle_{s.p.}$.

We notice, that a mean distance between levels D_l in the partial wave l and a characteristic reduced width γ_l of resonances are related by order of value [47]

$$\gamma_l \sim \frac{D_l}{\pi \mathfrak{a}_l R}. \quad (152)$$

Therefore

$$\frac{1}{N^{1/2}} \sim \left(\frac{D_l}{\omega}\right)^{1/2} \sim (\pi \mathfrak{a}_l R)^{1/2} \left(\frac{\gamma_l}{\omega}\right)^{1/2}. \quad (153)$$

We see, that an enhancement $\sim (\omega/\gamma_l)^{1/2}$ of the factors B'_l (117) in comparison with A'_l is exactly canceled by a depression $\sim (\gamma_l/\omega)^{1/2}$ of nondiagonal potentials $U_{P(PT)}^{(0\ 1p)}$ and $U_{P(PT)}^{(1s\ 0)}$ in comparison with a diagonal one $U_{P(PT)}^{(00)}$. This means, that the contributions (140), (141) from a mixing of functions of the elastic and inelastic channels either less, or considerably do not surpass the contribution (139) from a mixing in the elastic channel! Therefore we shall not discuss the components, which are proportional to $\delta S_{P(PT)}^{(0\ 1p)}$ and $\delta S_{P(PT)}^{(1s\ 0)}$.

We turn now to an analysis and comparison of contributions $\delta S_{P(PT)}^{(00)}$ and $\delta S_{P(PT)}^{(1s\ 1p)}$ to the observables. In the region far away from both mixing s- and p-wave resonances ($|\varepsilon - E_l| > \gamma_l^{1/2} \omega^{1/2}$), as we have just established (see Eqs.(123),

(124)), the components with amplitude A_l dominate in the wave function. In this region of purely potential scattering only the quantity

$$\left(\delta S_{P(PT)}^{(00)}\right)_{pot} \sim \xi_{P(PT)} \exp(i(\delta_s + \delta_p)) \frac{U_{P(PT)}^{(00)}}{\omega} (kR)^2 \quad (154)$$

is significant. We have taken into account, that $(A_l)_{pot} \simeq \exp(i\delta_l) s_l^{1/2}$. We note, that according to Eqs.(121) and (131) we are dealing here with the total phase shifts δ_l for elastic scattering. We get for P- and T-noninvariant observables

$$\frac{\epsilon_{P(PT)}^{(00)}}{n} \sim \frac{\chi_{P(PT)}^{(00)}}{n} \sim 4\pi R^2 \frac{U_{P(PT)}^{(00)}}{\omega}. \quad (155)$$

In the region $|\varepsilon - E_l| < \gamma_l^{1/2} \omega^{1/2}$ the factors B'_l (117) become considerable. We obtain the following expression for mixing amplitude in the inelastic channel (142)

$$\begin{aligned} \delta S_{P(PT)}^{(1s\ 1p)} &= \xi_{P(PT)} \exp(i(\delta_s^c + \delta_p^c)) \frac{U_{P(PT)}^{sp}}{\omega} \frac{\omega}{(\gamma_s \gamma_p)^{1/2}} (kR)^2 \times \\ &\times \frac{\gamma_s \gamma_p}{\left(\varepsilon - E_s + i s_0 \left(\gamma_s - \frac{\varepsilon - E_s}{\Phi_s^0}\right)\right) \left(\varepsilon - E_p + i s_1 \left(\gamma_p - \frac{\varepsilon - E_p}{\Phi_p^0}\right)\right)}, \end{aligned} \quad (156)$$

where

$$U_{P(PT)}^{sp} = U_{P(PT)}^{(1s\ 1p)} \frac{x_s x_p}{y_s y_p} \left(\frac{f_{P(PT)}(\mathfrak{a}'_s, \mathfrak{a}'_p)}{\mathfrak{a}'_s R \cos \mathfrak{a}'_s R \mathfrak{a}'_p R \sin \mathfrak{a}'_p R} \right)_{E_l}. \quad (157)$$

As far as $|E_s - E_p| \ll \omega$, it is unimportant, at what energy, E_s or E_p , the right part is taken here. A general expression for an amplitude of mixing in the elastic channel (139) is rather cumbersome. However, in some special interesting cases this general expression may be simplified.

We consider at first the region, lying near the resonances, but beyond their reduced widths ($\gamma_l < |\varepsilon - E_l| < \gamma_l^{1/2} \omega^{1/2}$). In particular, an interval between s- and p-wave resonances can belong to this region. The factors A_l still dominate over A'_l (see Eq.(123)). The estimates for them do not change, so the contributions from mixing in the elastic channel remain on the level of (154), (155). At the same time a mixing in the inelastic channel gives

$$\frac{\epsilon_{P(PT)}^{(1s\ 1p)}}{n} \sim \frac{\chi_{P(PT)}^{(1s\ 1p)}}{n} \sim 4\pi R^2 \frac{U_{P(PT)}^{sp}}{\omega} \frac{\omega \gamma_s^{1/2} \gamma_p^{1/2}}{(\varepsilon - E_s)(\varepsilon - E_p)}. \quad (158)$$

Though $\omega^{1/2} \gamma_l^{1/2} / |\varepsilon - E_l| > 1$, the potentials $U_{P(PT)}^{sp}$ are suppressed in comparison with $U_{P(PT)}^{(00)}$ by the factor of the scale of (153). An approximate evaluation shows,

that only in the region $\gamma_l < |\varepsilon - E_l| < \gamma_l^{3/4} \omega^{1/4}$ a mixing in the inelastic channel provides an increase of P- and T-noninvariant effects over the background values (155).

We turn now to an analysis of nearest vicinities of resonances. We assume, that a distance between s- and p-wave resonances surpasses their reduced widths. To be more specific,

$$\gamma_s |\Phi_s^0| \sim \gamma_p |\Phi_p^0| \ll |E_s - E_p|, \quad (159)$$

so the condition (123) may be used. Consequently near p-wave resonance ($|\varepsilon - E_p| \ll \gamma_p |\Phi_p^0|$) we have $|A'_s| \ll |A_s|$. We get for P- (or P-,T-) noninvariant amplitude of mixing in the elastic channel

$$\delta S_{P(PT)}^{(00)} \simeq \xi_{P(PT)} \exp(i(\delta_s + \delta_p^c)) \frac{U_{P(PT)}^p(kR)^2}{\omega} \frac{\gamma_p}{\varepsilon - E_p + i\frac{\Gamma_p}{2}}. \quad (160)$$

Similarly, near s-wave resonance ($|\varepsilon - E_s| \ll \gamma_s |\Phi_s^0|$) $|A'_p| \ll |A_p|$, so an amplitude of interest is of the form

$$\delta S_{P(PT)}^{(00)} \simeq \xi_{P(PT)} \exp(i(\delta_s^c + \delta_p)) \frac{U_{P(PT)}^s(kR)^2}{\omega} \frac{\gamma_s}{\varepsilon - E_s + i\frac{\Gamma_s}{2}}. \quad (161)$$

For simplification we use designations for the depths of P- and T-noninvariant potentials

$$U_{P(PT)}^p = U_{P(PT)}^{(00)} x_p \left(\frac{1}{\varepsilon_s R \cos \varepsilon_s R} \right)_{E_s} \left(\frac{f_{P(PT)}(\varepsilon_s, \varepsilon'_p)}{\varepsilon'_p R \sin \varepsilon'_p R} - \frac{f_{P(PT)}(\varepsilon_s, \varepsilon_p)}{\varepsilon_p R \sin \varepsilon_p R} \right)_{E_p}, \quad (162)$$

$$U_{P(PT)}^s = U_{P(PT)}^{(00)} x_s \left(\frac{1}{\varepsilon_p R \cos \varepsilon_p R} \right)_{E_p} \left(\frac{f_{P(PT)}(\varepsilon'_s, \varepsilon_p)}{\varepsilon'_s R \cos \varepsilon'_s R} - \frac{f_{P(PT)}(\varepsilon_s, \varepsilon_p)}{\varepsilon_s R \cos \varepsilon_s R} \right)_{E_s}, \quad (163)$$

just as in Eqs.(156), (157).

Let us compare the amplitudes in resonances with the background value of (154). If $\varepsilon = E_l$, the amplitudes (160) and (161) of s- and p-wave mixing in the elastic channel surpass the potential value (154) by the factor of $\sim 1/s_l$. The reason is that in the potential scattering a jump of potential and centrifugal barrier hinder a penetration of a neutron into a nucleus, while in the resonance these factors hinder an exit of a neutron from a nucleus, increasing an interaction time and, therefore, a mixing amplitude (see Eqs.(126), (127) and (129)). We consider now the amplitude of s- and p-wave mixing in the inelastic channel (156). In the resonance ($\varepsilon = E_l$) it is enhanced by the factor ω/Γ_l . According to Eq.(130) this quantity corresponds to the delay of a neutron inside a nucleus due to a weak coupling between the channels. However, account must

be taken of the suppression of the depth of P- (P-,T-) noninvariant potential $U_{P(PT)}^{sp}$ in comparison with $U_{P(PT)}^{(00)}$ by the factor of $1/N^{1/2} \sim (\gamma_l/\omega)^{1/2}$. Nevertheless, multiplying these factors, we obtain in the inelastic channel an enhancement $(1/s_l^{1/2})(\omega/\Gamma_l)^{1/2} \sim (1/s_l)(\omega/\gamma_l)^{1/2}$. This quantity surpasses an enhancement in the elastic channel by the factor of $\sim (\omega/\gamma_l)^{1/2}$!

Thus, far away from s- and p-wave resonances ($|\varepsilon - E_l| > \gamma_l^{3/4} \omega^{1/4}$) P- and T-noninvariant effects are determined by a mixing in the elastic channel (by the quantity $\delta S_{P(PT)}^{(00)}$). In a resonance the quantities $\delta S_{P(PT)}^{(00)}$ and $\delta S_{P(PT)}^{(1s\ 1p)}$ peak. However, $\delta S_{P(PT)}^{(1s\ 1p)}$ surpasses $\delta S_{P(PT)}^{(00)}$ by the factor of $\sim (\omega/\gamma_l)^{1/2}$. Therefore, P- and T-noninvariant effects in resonances result mainly from a mixing in the inelastic channel.

For the first time an existence of an enhancement factor $(\omega/\Gamma)^{1/2}$ in the region of compound resonances was mentioned in Refs.[29, 30] in connection with an analysis of sensitivity of detailed balance tests to T-invariance violation. In resonances the time of neutron-nucleus interaction increases; so the P- and T-noninvariant mixing amplitudes are enhanced. In such context the form $(\omega/\Gamma)^{1/2}$ of an enhancement factor seems natural.

However, the other form of an enhancement factor is usually used. One separates the ratio $U_{P(PT)}^{sp}/(E_p - E_s)$ in the amplitude of mixing in the inelastic channel (156). This quantity is interpreted as the amplitude of P- or P-,T-noninvariant mixing of s-wave resonance to p-wave one or vice versa. As far as $|E_p - E_s| \sim D \sim \omega/N$, and $U_{P(PT)}^{sp} \sim U_{P(PT)}^{(00)}/N^{1/2}$, we have

$$\frac{U_{P(PT)}^{sp}}{|E_p - E_s|} \sim N^{1/2} \frac{U_{P(PT)}^{(00)}}{\omega}. \quad (164)$$

Such enhancement is said to be dynamic. It is related with a proximity of mixing resonances. For the first time it was analyzed in Refs.[19]-[21].

The amplitude of s- and p-wave mixing in compound resonance is usually taken in the form

$$\delta S_{P(PT)}^{sp} = \frac{\xi_{P(PT)}}{2} \exp(i(\delta_s^c + \delta_p^c)) U_{P(PT)}^{sp} \frac{(\Gamma_s^n)^{1/2} (\Gamma_p^n)^{1/2}}{\left(\varepsilon - E_s + i\frac{\Gamma_s}{2}\right) \left(\varepsilon - E_p + i\frac{\Gamma_p}{2}\right)}. \quad (165)$$

With regard to coincidence in the model of neutron width, Γ_l^n , and total width, Γ_l , this expression is very similar to Eq.(156), but is not identical to it. The reason is that the denominators $[l]$ (120), entering in the formula (156), have usual Breit-Wigner form only in the vicinities of resonance energies, which are small in comparison with reduced widths. As far as the distances between s- and p-wave resonances are usually comparable or surpass the reduced widths of resonances, the denominator $[s]$ has not the Breit-Wigner form near p-wave

resonance, while a similar deviation takes place for the quantity $[p]$ near s-wave resonance. These deviations are, however, rather simple. If $|\varepsilon - E_l| \gg \gamma_l |\Phi_l^0|$, then we get according to Eq.(131)

$$\exp(i\delta_l^c) \frac{1}{[l]} \simeq \exp(i\delta_l) \frac{1}{\varepsilon - E_l}. \quad (166)$$

Thus, the amplitude of mixing in the inelastic channel (156) near p-wave resonance is of the form

$$\delta S_{P(PT)}^{(1s\ 1p)} \simeq \frac{\xi_{P(PT)}}{2} \exp(i(\delta_s^c + \delta_p^c)) U_{P(PT)}^{sp} \frac{(\Gamma_s^n)^{1/2} (\Gamma_p^n)^{1/2}}{(\varepsilon - E_s) \left(\varepsilon - E_p + i s_1 \left(\gamma_p - \frac{\varepsilon - E_p}{\Phi_p^0} \right) \right)}, \quad (167)$$

while near s-wave resonance we obtain

$$\delta S_{P(PT)}^{(1s\ 1p)} \simeq \frac{\xi_{P(PT)}}{2} \exp(i(\delta_s^c + \delta_p)) U_{P(PT)}^{sp} \frac{(\Gamma_s^n)^{1/2} (\Gamma_p^n)^{1/2}}{\left(\varepsilon - E_s + i s_0 \left(\gamma_s - \frac{\varepsilon - E_s}{\Phi_s^0} \right) \right) (\varepsilon - E_p)}. \quad (168)$$

General expressions for the energy dependent P- and T-noninvariant observables (133), (134), caused by a mixing in the inelastic channel, are rather cumbersome. However, the situation is simplified close to resonance energies due to approximate relationships (167), (168). Near p-wave resonance we have with an accuracy of $(kR)^2$

$$\frac{\epsilon_{P(PT)}^p}{n} = -\nu_{P(PT)} \frac{2\pi}{k^2} g_J U_{P(PT)}^{sp} \frac{kR \left(1 - \frac{1}{\Phi_s^0} \right) (\Gamma_s^n)^{1/2} (\Gamma_p^n)^{1/2} \left(\varepsilon - E_p + \frac{(kR)^2 \gamma_p}{1 - \frac{1}{\Phi_s^0}} \right)}{(\varepsilon - E_s) \left((\varepsilon - E_p)^2 + (s_1)^2 \left(\gamma_p - \frac{\varepsilon - E_p}{\Phi_p^0} \right)^2 \right)}, \quad (169)$$

$$\frac{\chi_{P(PT)}^p}{n} = \nu_{P(PT)} \frac{2\pi}{k^2} g_J U_{P(PT)}^{sp} \frac{(\Gamma_s^n)^{1/2} (\Gamma_p^n)^{1/2} (\varepsilon - E_p)}{(\varepsilon - E_s) \left((\varepsilon - E_p)^2 + (s_1)^2 \left(\gamma_p - \frac{\varepsilon - E_p}{\Phi_p^0} \right)^2 \right)}, \quad (170)$$

where $\nu_P = 1$, $\nu_{PT} = -C_{J\frac{1}{2}}/g_J$. Similarly near s-wave resonance P- and T-

noninvariant observables take the form

$$\frac{\epsilon_{P(PT)}^s}{n} = -\nu_{P(PT)} \frac{2\pi}{k^2} g_J U_{P(PT)}^{sp} \frac{kR \left(1 - \frac{1}{\Phi_s^0}\right) (\Gamma_s^n)^{1/2} (\Gamma_p^n)^{1/2} \left(\varepsilon - E_s + \frac{\gamma_s}{1 - \frac{1}{\Phi_s^0}}\right)}{(\varepsilon - E_p) \left((\varepsilon - E_s)^2 + (s_0)^2 \left(\gamma_s - \frac{\varepsilon - E_s}{\Phi_s^0}\right)^2\right)}, \quad (171)$$

$$\frac{\chi_{P(PT)}^s}{n} = \nu_{P(PT)} \frac{2\pi}{k^2} g_J U_{P(PT)}^{sp} \frac{(\Gamma_s^n)^{1/2} (\Gamma_p^n)^{1/2} (\varepsilon - E_s - (kR)^2 \gamma_s)}{(\varepsilon - E_p) \left((\varepsilon - E_s)^2 + (s_0)^2 \left(\gamma_s - \frac{\varepsilon - E_s}{\Phi_s^0}\right)^2\right)}. \quad (172)$$

We see, that in the vicinities of resonances the transmission asymmetries $\epsilon_{P(PT)}^l$ and angles of spin rotation $\chi_{P(PT)}^l$ cross zero. However, it is easy to establish, that in the intervals of energy

$$|\varepsilon - E_l| \leq \frac{\Gamma_l}{2} = (kR)^{2l+1} \gamma_l \quad (173)$$

the quantities $\epsilon_{P(PT)}^l$ peak and may be presented in the form

$$\frac{\epsilon_{P(PT)}^l}{n} = -\mathcal{P}_{P(PT)}^l(\varepsilon) \sigma_l(\varepsilon). \quad (174)$$

Here $\sigma_l(\varepsilon)$ is a partial cross section of neutron-nucleus interaction

$$\sigma_l(\varepsilon) = \frac{\pi}{k^2} g_J \frac{\Gamma_l^n \Gamma_l}{(\varepsilon - E_l)^2 + \left(\frac{\Gamma_l}{2}\right)^2}, \quad (175)$$

while factors $\mathcal{P}_{P(PT)}^l(\varepsilon)$ are slow functions of energy

$$\mathcal{P}_{P(PT)}^p(\varepsilon) = \nu_{P(PT)} \frac{U_{P(PT)}^{sp}}{\varepsilon - E_s} \left(\frac{\Gamma_s^n}{\Gamma_p^n}\right)^{1/2}, \quad (176)$$

$$\mathcal{P}_{P(PT)}^s(\varepsilon) = \nu_{P(PT)} \frac{U_{P(PT)}^{sp}}{\varepsilon - E_p} \left(\frac{\Gamma_p^n}{\Gamma_s^n}\right)^{1/2}. \quad (177)$$

Transmission asymmetry $\epsilon_{P(PT)}^l$ crosses zero outside an interval (173). While an angle $\chi_{P(PT)}^l$ crosses zero within the width of resonance Γ_l . Neglecting a shift $(kR)^2 \gamma_s$ in comparison with a width $\Gamma_s/2 = (kR) \gamma_s$, we obtain for the quantities $\chi_{P(PT)}^l$ in the intervals (173)

$$\chi_{P(PT)}^l = -\frac{2(\varepsilon - E_l)}{\Gamma_l} \epsilon_{P(PT)}^l. \quad (178)$$

The resonance formulas (174)-(178) coincide with those found by using a mixing amplitude in the standard form (165).

In experiments one chooses a thickness of a target to provide an optimum rate of statistic gathering. In a resonance $\varepsilon \sim E_l$ this condition gives (see, for example, Ref.[11])

$$n \sim \frac{1}{\sigma_l(E_l)}. \quad (179)$$

It follows that the quantities $\mathcal{P}_{P(PT)}^l(E_l)$ approximately equal to measured asymmetries $\epsilon_{P(PT)}^l$ at the energies $\varepsilon = E_l$ and angles $\chi_{P(PT)}^l$ at the energies $\varepsilon = E_l \pm \Gamma_l/2$.

We see now, that in a p-wave resonance a measured quantity (176) is enhanced by a factor of $(\Gamma_s^n/\Gamma_p^n)^{1/2} \sim 1/kR$ besides the factor (164). This enhancement is said to be structural or kinematic. While in a s-wave resonance (see Eq.(177)) a factor of suppression $(\Gamma_p^n/\Gamma_s^n)^{1/2} \sim kR$ arises. We remind, that in the model considered a total width of resonance Γ_l coincides with its neutron width Γ_l^n . This situation approximately corresponds to compound resonances in light nuclei. Usually the Eqs.(174)-(178) are analyzed as applied to heavy nuclei, where $\Gamma_l \gg \Gamma_l^n$, and $\Gamma_s \sim \Gamma_p$. Let us show, that the additional factors of enhancement or suppression, arising in Eqs.(176) and (177), have different meaning for light and heavy nuclei.

Let $\Gamma_s = \Gamma_s^n \gg \Gamma_p = \Gamma_p^n$. Thus it is easy to see, that s- and p-wave resonance cross sections (175) reach approximately equal maximal values. Therefore, according to condition (179) measurements in s- and p-wave resonances should be carried on targets with the same thickness. But as far as the quantity $\mathcal{P}_{P(PT)}^p(E_p)$ is enhanced in comparison with $\mathcal{P}_{P(PT)}^s(E_s)$ by a factor of $\sim 1/(kR)^2$, the P- and T-noninvariant observables (174), (178) will be a $\sim 1/(kR)^2$ times more in the p-wave resonance than in the s-wave one. It has been just this result that was obtained earlier in the model. Indeed, an enhancement factor of the amplitude $\delta S_{P(PT)}^{(1s\ 1p)}$ in a resonance in the partial wave l was found equal $(1/s_l)(\omega/\gamma_l)^{1/2}$. Clearly, this factor is a $s_0/s_1 \sim 1/(kR)^2$ times more in the p-wave than in the s-wave.

We consider now a case $\Gamma_s \sim \Gamma_p \gg \Gamma_s^n \gg \Gamma_p^n$. A resonance cross section $\sigma_s(E_s)$ surpasses now $\sigma_p(E_p)$ by a factor of $\sim 1/(kR)^2$. But the quantity $\mathcal{P}_{P(PT)}^s(E_s)$ is a $\sim 1/(kR)^2$ times less than $\mathcal{P}_{P(PT)}^p(E_p)$. Therefore, in heavy nuclei one should expect the same P- and T-noninvariant effects (174), (178) in the s- and p-wave resonances, if the targets with equal thickness are used. However, in view of condition (179), in the p-wave resonance measurement a target should be used a $\sim 1/(kR)^2$ times thicker, than in the s-wave resonance measurement. Clearly, in this situation the observables in the p-wave resonance appear a $\sim 1/(kR)^2$ times more, than in the s-wave resonance.

Thus, in light nuclei an additional enhancement of P- and T-noninvariant

effects in p-wave resonance by a factor of $(\Gamma_s^n/\Gamma_p^n)^{1/2} \sim 1/kR$ results from an increase of neutron delay inside a nucleus. In heavy nuclei the total widths of s- and p-wave resonances are determined by radiative transitions, therefore the life times of these resonances are approximately equal. So an additional enhancement in the p-wave resonance, having the same value $(\Gamma_s^n/\Gamma_p^n)^{1/2} \sim 1/kR$ that in light nuclei, arises rather from the advantages of use of thick target.

7 Model calculation of P- and T-noninvariant effects

In the previous section we have shown that the inelastic channel contributes decisively to the mixing of s- and p-waves. Within the widths of resonances we have got the usual expressions (174)–(178) for P- and T-noninvariant observables. As an illustration let us present the results of exact calculation of P- and T-noninvariant effects (133), (134) for a close-lying pair of s- and p-wave resonances.

As far as the model does not account radiative channels, it seems reasonable to use it for description of resonances in light nuclei. We take as an example a pair of s- and p-wave resonances of the ^{35}Cl nucleus with energies $E_s = 26.60$ keV and $E_p = 22.41$ keV and neutron widths $2g\Gamma_s^n = 130$ eV and $2g\Gamma_p^n = 4$ eV [48]. Radiative contributions to these widths are ~ 0.5 eV only; we neglect them. The nucleus ^{35}Cl has spin and parity $I^\pi = 3/2^+$. We assume that spins of the resonances chosen equal $J = 1$ (there is no information about these spins in Ref.[48]).

We remind, that the partial wave is specified by three quantum numbers l , j and J . We restrict our attention to two partial waves $l = 0$, $j = 1/2$, $J = 1$ and $l = 1$, $j = 1/2$, $J = 1$. For brevity we shall label the quantities, associated with these waves, only by indexes s and p.

Neutron-nucleus potential in the elastic and inelastic channels is taken to be spherical potential well of the radius $R = 6.5$ fm and depth

$$U = U_0 + U_{ls} \hat{l} \hat{s}, \quad (180)$$

where $U_0 = 23$ MeV and $U_{ls} = 1$ MeV. So the parameters of potential (77) for s1/2- and p1/2-waves equal $U_s = 23$ MeV and $U_p = 22$ MeV. Characteristic one-particle energy (75) is $\omega = 490.444$ keV. The sequence of bound states in the potential chosen is presented in Fig.1. We assume that 18 neutrons of ^{35}Cl nucleus fill the levels 1s1/2, 1p3/2, 1p1/2, 1d5/2 and 1d3/2 so the states 2s1/2 and 2p1/2 are free. In the framework of the model, described in the section 5, s- and p-wave resonances correspond to transitions of the incident neutron to bound states 2s1/2 and 2p1/2, respectively, accompanied by excitation of the target to the levels with energies ε_s and ε_p .

Using Eq.(105) we have found the reduced widths of resonances $\gamma_s = 372$ eV and $\gamma_p = 285$ eV. We have in each partial wave the set of two equations

$$\begin{cases} \Phi_l(E_l) = 0, \\ \frac{d}{d\varepsilon}\Phi_l(\varepsilon)\big|_{\varepsilon=E_l} = -\frac{1}{\gamma_l}, \end{cases} \quad (181)$$

that gives us two parameters of the model: ε_l and W_l . Solving these sets we obtain $\varepsilon_s = 8831.277$ keV, $W_s = 174.509$ keV and $\varepsilon_p = 1589.982$ keV, $W_p = 54.422$ keV. We notice, that dimensionless parameter λ_l takes here the values $\lambda_s = 0.020$ and $\lambda_p = 0.034$, so we are dealing with the case of weak coupling between channels ($\lambda_l \ll 1$).

A weak coupling approximation was studied in the section 5.2. Its quality may be checked by calculation of reduced widths using Eq.(112). We get then for the s-wave width 372 eV, differing from the exact value only in the first decimal, while for the p-wave width — 294 eV, differing from the exact value by 3 %.

In Fig.2 the energy dependencies of logarithmic derivatives $\Phi_l(\varepsilon)$ (106) are shown. Their forms agree well with Eq.(111). It is seen that the Breit-Wigner approximation (103) holds only in a small vicinity of each resonance. Far from the resonances the functions $\Phi_l(\varepsilon)$ reach the potential values (107), which equal $\Phi_s^0 = 10.706$ and $\Phi_p^0 = -3.188$.

Using Eq.(98) it is easy to calculate the phase shifts for each partial wave

$$\delta_l(\varepsilon) = \delta_l^c(\varepsilon) + \arctg \frac{s_l(\varepsilon)}{\Phi_l(\varepsilon)}, \quad (182)$$

and cross sections

$$\sigma_l(\varepsilon) = \frac{4\pi}{k^2} g_J \sin^2 \delta_l(\varepsilon). \quad (183)$$

They are presented in Fig.3. We have for these cross sections at the resonance energies $\sigma_s(E_s) = 36.681$ b and $\sigma_p(E_p) = 43.572$ b.

The amplitudes A_l and A'_l (see Eqs.(96)) are complex. However, as noted above (see text before Eq.(131)), at $\varepsilon < \varepsilon_l$ the phases of these factors coincide with the phase shift δ_l . Let $A_l = \alpha_l \exp(i\delta_l)$ and $A'_l = \alpha'_l \exp(i\delta_l)$. In Fig.4 the real quantities α_l and α'_l are shown versus the energy. According to the Eqs.(115), (116) and (127) an enhancement of the factors A_l and A'_l is $\sim 1/kR$ times as large for the p-wave resonance as for the s-wave one. We have here $k(E_s)R = 0.233$ and $k(E_p)R = 0.214$, so the enhancement is on the scale of $\sim 4 - 5$. An increase of the amplitudes near the resonances is displayed in Figs.4a and 4b. Fig.4c shows that the factors A_l fall more slowly than A'_l (see Eq.(123)). According to Eqs.(86) to obtain the factors B'_s and B'_p we multiply A'_s and A'_p by $-1/\lambda_s = -50.626$ and $-1/\lambda_p = -29.250$, respectively.

We turn now to estimates for P- and T-noninvariant observables (133) and (134). We restrict our attention to the dominating contribution $\delta S_{P(PT)}^{(1s\ 1p)}$ to the sum (138). Let

$$U_P^{(00)} = 10^{-1} \text{ eV}, \quad U_{PT}^{(00)} = 10^{-4} \text{ eV}, \quad (184)$$

so an unknown constant ϕ is taken to be 10^{-3} . According to Eqs.(151) and (153) we take as the depth of a nondiagonal weak potential the quantity

$$U_P^{(1s\ 1p)} = \left(\frac{\pi \mathfrak{a}_s R \gamma_s}{\omega} \right)_{E_s}^{1/2} U_P^{(00)} = 1.3 \cdot 10^{-2} \text{ eV}, \quad (185)$$

similarly

$$U_{PT}^{(1s\ 1p)} = 1.3 \cdot 10^{-5} \text{ eV}. \quad (186)$$

It is easy to estimate, that the potential values (155) of P-noninvariant observables are thus on the scale of $\sim 10^{-6}$ b, while for P-,T-noninvariant ones we get $\sim 10^{-9}$ b.

Taking into account the identity of the phases of the factors B'_s and B'_p and the phase shifts for elastic scattering and putting $B'_l = \beta'_l \exp(i\delta_l)$, we obtain from Eq.(142) for the observables

$$\frac{\epsilon_{P(PT)}}{n} = \nu_{P(PT)} \frac{4\pi}{k^2} g_J \sin(\delta_s + \delta_p) \frac{U_{P(PT)}^{(1s\ 1p)}}{\omega} \beta'_s \beta'_p f_{P(PT)}(\mathfrak{a}'_s, \mathfrak{a}'_p), \quad (187)$$

$$\frac{\chi_{P(PT)}}{n} = \nu_{P(PT)} \frac{4\pi}{k^2} g_J \cos(\delta_s + \delta_p) \frac{U_{P(PT)}^{(1s\ 1p)}}{\omega} \beta'_s \beta'_p f_{P(PT)}(\mathfrak{a}'_s, \mathfrak{a}'_p). \quad (188)$$

We notice, that for the case considered here $J = I - 1/2$ we have $C_{J\frac{1}{2}} = g_J$, so $\nu_P = 1$, and $\nu_{PT} = -1$. The integrals $f_{P(PT)}(\mathfrak{a}'_s, \mathfrak{a}'_p)$ depend only slightly on the energy (they belong to the functions, that change significantly only on the scale of ω). At $\varepsilon = E_s$ these integrals equal $f_P(\mathfrak{a}'_s, \mathfrak{a}'_p) = 4.892$ and $f_{PT}(\mathfrak{a}'_s, \mathfrak{a}'_p) = 0.522$.

In Figs.5 and 6 the asymmetries $\epsilon_{P(PT)}/n$ and angles $\chi_{P(PT)}/n$ are presented versus the energy. Figs.5a, 6a and 5b, 6b show the effects near p- and s-wave resonances, respectively. Within the widths of the resonances these curves are described well by Eqs.(174)-(178). In particular, the angles of spin rotation cross zero at $\varepsilon \simeq E_l$. In the same time in accordance with Eqs.(169), (171) the transmission asymmetries cross zero at the energies $\varepsilon \simeq E_l - \Gamma_l/(2kR(1 - 1/\Phi_s^0))$, that is before the resonances beyond their widths. Figs.5c, 6c display it.

We see that the effects are maximal in the p-wave resonance. The asymmetry ϵ_P/n reach the value $\sim 4 \cdot 10^{-2}$ b, which is $\sim 4 \cdot 10^4$ times greater than the potential value! An enhancement factor involves $(\omega/\gamma_p)^{1/2} \sim 40$, $1/(kR)^3 \sim 10^2$ and numerical coefficients like $f_P(\mathfrak{a}'_s, \mathfrak{a}'_p)$. While we get $\sim 4 \cdot 10^{-6}$ b for the maximal value of ϵ_{PT}/n , which surpasses the potential estimate by a factor $\sim 4 \cdot 10^3$ only.

The reason is that the integral of the overlap $f_{PT}(\mathfrak{a}'_s, \mathfrak{a}'_p)$ is found one order lower than $f_P(\mathfrak{a}'_s, \mathfrak{a}'_p)$. Clearly, the energy dependencies of P- and P-,T-noninvariant observables are the same. So the curves in Figs.5 and 6 differ only in sign and factor of $\sim 10^{-4}$. So in what follows we discuss the P-nonconserving effects only.

In the resonance measurements the target thicknesses should be taken to be $n \sim 1/\sigma_l(E_l) \sim 0.02 \text{ b}^{-1}$. In the p-wave resonance we obtain for P-noninvariant asymmetry ϵ_P and angle χ_P an estimate $\sim 10^{-3}$ (for P-,T-noninvariant observables ϵ_{PT} and $\chi_{PT} \sim 10^{-7}$). In the s-wave resonance these quantities are $1/(kR)^2 \sim 25$ times suppressed.

It is interesting, that there exist the regions beyond the resonance widths, which are favorable for measurements. This is because the quantities $\epsilon_{P(PT)}$ and $\chi_{P(PT)}$ fall more slowly than the cross sections. Fig.7 shows, that to the right and to the left of the p-wave resonance there are the energy intervals, where the cross section equals $\sim 1 \text{ b}$, while $\epsilon_P/n \sim 0.5 \cdot 10^{-3} \text{ b}$ and $\chi_P/n \sim 10^{-3} \text{ b}$. On these intervals to provide the optimum rate of statistic gathering the target thickness should be equal to $n \sim 1 \text{ b}^{-1}$, so the observables ϵ_P and χ_P are of the same scale $\sim 10^{-3}$, than in the p-wave resonance. We notice, that a total s-wave cross section near p-wave resonance is a sum of contributions of partial waves $l = 0, j = 1/2, J = 1$ and $l = 0, j = 1/2, J = 2$. The latter was omitted by us. However, these contributions are, obviously, comparable. So our conclusions remain valid.

A completely different type of situation occurs near an interference minimum to the left of s-wave resonance. In the case considered, when a target spin I differs from zero, there are no deep minimums in a total cross section owing to superposition of the contributions from waves with $J = I \pm 1/2$. We remind, that the P-,T-noninvariant correlation ($\sigma[\mathbf{n}_k \mathbf{n}_I]$) could be studied only on the targets with nonzero spin I . However, one may observe P-nonconserving effects in the interaction of neutrons with spinless target nuclei. In this case only one partial wave $l = 0, j = J = 1/2$ contributes to s-wave cross section. Therefore, let us consider a vicinity of an interference minimum, keeping in mind the possible applications to the case of neutron interaction with spinless nuclei.

In Fig.8 the calculated quantities ϵ_P/n , χ_P/n and $\sigma(\varepsilon) = \sigma_s(\varepsilon) + \sigma_p(\varepsilon)$ are presented. In accordance with Fig.3 a minimum of cross section is determined by p-wave scattering. In fact a radiative capture is of first importance in the minimum. It is easy to estimate a contribution from the nearest s-wave resonance, taking its radiative width to be $\Gamma_s^\gamma = 0.5 \text{ eV}$. A capture cross section

$$\sigma_s^\gamma(\varepsilon) = \frac{\pi}{k^2} g_J \frac{\Gamma_s^n \Gamma_s^\gamma}{(\varepsilon - E_s)^2 + \left(\frac{\Gamma_s}{2}\right)^2} \quad (189)$$

is found to be 0.013 b in the minimum of the scattering cross section at $\varepsilon \simeq 26.2 \text{ keV}$. We notice now, that for the value $\sim 0.1 \text{ b}$ of the total cross section a target with thickness $n \sim 10 \text{ b}^{-1}$ gives the optimum rate of statistic gathering.

So the measuring angle of spin rotation in the region of an interference minimum may have the magnitude $\chi_P \sim 10^{-3}$, which is comparable with the effect in the p-wave resonance! A situation with an asymmetry is worse as this quantity crosses zero in the interval considered.

8 Summary

The paper deals with a simplified model for resonant neutron-nucleus interaction based on the scheme of coupled channels. The equations and boundary conditions for radial functions are written in section 2. The form of diagonal and nondiagonal potentials, as well as a number of channels are external parameters of the model.

Perturbations violating P- and T-invariance are weak enough to be taken into account in the first order. The formulas for P- and T-noninvariant corrections to S-matrix are presented in section 4. The single-particle operators of weak interaction of the form $\sigma \mathbf{p}$ and $\sigma \mathbf{r}$ are considered in details.

The analytical solution for the problem with two coupled channels and square-well potentials is given in section 5. A neutron orbital momentum l is arbitrary. The case of a weak channel coupling corresponding to narrow Breit-Wigner resonance is analyzed.

A mixing of two narrow s- and p-wave resonances by P- and P-,T-noninvariant potentials is studied in section 6. In the framework of the model the contributions of four types to the mixing amplitude are separated. Here, we are dealing with transitions from s- to p-wave (or vice versa) in the elastic channel, inelastic channel, as well as with the cross terms. It is shown, that the cross terms do not exceed the contribution from a mixing in the elastic channel. An enhancement of the mixing amplitude in the elastic channel in a resonance with an orbital momentum l is of the scale $\sim 1/(kR)^{2l+1}$, while that in the inelastic channel reaches $\sim (\omega/\gamma_l)^{1/2}(1/(kR)^{2l+1})$. Here ω is the characteristic one-particle energy and γ_l is the reduced width of the resonance. Thus, the contribution from a mixing in the inelastic channel dominates.

The analytical expression for the amplitude of mixing in the inelastic channel differs slightly from the usually used formula for compound-compound mixing. The reason is the resonant amplitudes deviate from the Breit-Wigner energy dependence beyond the widths of resonances. Nevertheless we obtain the usual expressions for P- and T-noninvariant observables within the widths of resonances. As far as the total width of a resonance coincides with the neutron width, the model is directly appropriate for light nuclei only. In this situation an expansion of the factor of resonance enhancement into dynamic and structural (kinematic) ones is conventional.

A close-lying pair of s- and p-wave resonances of the nucleus ^{35}Cl is reproduced in section 7. We emphasize, that our description of neutron-nucleus interaction is

very schematic. A resonance enhancement of P- and T-noninvariant observables is demonstrated. The favorable possibilities are shown to exist on thick targets for measurements beyond the resonance widths. In particular, an interference minimum near s-wave resonance is of interest for P-odd neutron spin rotation on light spinless nuclei.

An including of radiative channels will allow to analyze P- and T-noninvariant effects, first, for heavy nuclei, secondly, in radiative neutron capture. On the other hand, an increase of a number of coupled channels will lead to large sets of s- and p-wave resonances. It may be significant for an analysis of the sign correlation of P-odd effects [13]. A replacement of square-well potentials by potentials of Woods-Saxon type, which are convenient for numerical solution of coupled equations, will make the model more realistic.

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Appendix

The numerical factors A, B, and C enter into the amplitude of elastic scattering of s- and p-wave neutrons by polarized nuclei (see Eqs.(20), (23)-(30)). They are expressed in terms of the normalized Racah functions and 9j-simbols

$$A_J^{(1)} = -g_J \left(\frac{3I}{I+1} \right)^{1/2} U(J \frac{1}{2} I 1, I \frac{1}{2}), \quad (A1)$$

$$A_{Jjj'}^{(2)} = -g_J \left(\frac{3I}{I+1} \right)^{1/2} U(JjI1, Ij') U(1 \frac{1}{2} j' 1, j \frac{1}{2}), \quad (A2)$$

$$A_{Jjj'}^{(3)} = -3g_J \left(\frac{6I}{I+1} \right)^{1/2} (2j+1)^{1/2} U(JjI1, Ij') \begin{Bmatrix} j' & 1 & 1/2 \\ j & 1 & 1/2 \\ 1 & 2 & 1 \end{Bmatrix}, \quad (A3)$$

$$B_{Jj} = (-1)^{3/2-j} g_J \left(\frac{3I}{I+1} \right)^{1/2} U(J \frac{1}{2} I 1, Ij), \quad (A4)$$

$$C_{Jj} = 3(-1)^{j-1/2} g_J \left(\frac{I}{2(I+1)} \right)^{1/2} U(J \frac{1}{2} I 1, Ij) U(\frac{1}{2} 1 j 1, \frac{1}{2} 1). \quad (A5)$$

Using the explicit expressions for Racah functions [45] and 9j-simbols [49], we get

$$A_J^{(1)} = -3A_{J\frac{1}{2}\frac{1}{2}}^{(2)} = \frac{3}{2}A_{J\frac{1}{2}\frac{1}{2}}^{(3)} = B_{J\frac{1}{2}} = -C_{J\frac{1}{2}}, \quad (A6)$$

$$A_{J\frac{1}{2}\frac{3}{2}}^{(2)} = A_{J\frac{3}{2}\frac{1}{2}}^{(2)} = 4A_{J\frac{1}{2}\frac{3}{2}}^{(3)} = 4A_{J\frac{3}{2}\frac{1}{2}}^{(3)} = \frac{2}{3}B_{J\frac{3}{2}} = \frac{4}{3}C_{J\frac{3}{2}}, \quad (A7)$$

$$A_{J\frac{3}{2}\frac{3}{2}}^{(2)} = -5A_{J\frac{3}{2}\frac{3}{2}}^{(2)}, \quad (A8)$$

where

J	$I - \frac{1}{2}$		$I + \frac{1}{2}$
	$I = \frac{1}{2}$	$I > \frac{1}{2}$	
$A_J^{(1)}$	$-\frac{I}{2I+1}$		$\frac{I}{2I+1}$
$A_{J\frac{1}{2}\frac{3}{2}}^{(2)}$	$\frac{2I}{3(2I+1)} \left(\frac{2I-1}{I+1} \right)^{1/2}$		$\frac{2I}{3(2I+1)} \left(\frac{2I+3}{I} \right)^{1/2}$
$A_{J\frac{3}{2}\frac{3}{2}}^{(2)}$	0	$-\frac{I(I+4)}{3(2I+1)(I+1)}$	$\frac{I-3}{3(2I+1)}$

J	$I - \frac{3}{2}$	$I + \frac{3}{2}$
$A_{J\frac{3}{2}\frac{3}{2}}^{(2)}$	$-\frac{I-1}{2I+1}$	$\frac{I(I+2)}{(2I+1)(I+1)}$

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FIGURE CAPTIONS

Fig.1. Sequence of levels for the spherical square well of the radius 6.5 fm. The depth of the well is given by Eq.(180). Dashed lines correspond to the bottom levels, on which 18 neutrons locate. Dotted lines show the free s- and p-levels.

Fig.2. Logarithmic derivatives (106) of the s- and p-wave functions of the elastic channel versus the energy. Solid line — $\Phi_s(\varepsilon)$, dashed line — $\Phi_p(\varepsilon)$.

Fig.3. Partial cross sections (183) of s- and p-wave scattering versus the energy. Solid line — $\sigma_s(\varepsilon)$, dashed line — $\sigma_p(\varepsilon)$.

Fig.4. Amplitudes of the s- and p-wave functions of the elastic channel (79) near p-wave resonance (a), near s-wave resonance (b), in a wide range of energy (c). Solid line — $\alpha_s(\varepsilon)$, dashed line — $\alpha'_s(\varepsilon)$, dotted line — $\alpha_p(\varepsilon)$, dash-dotted line — $\alpha'_p(\varepsilon)$.

Fig.5. P-noninvariant transmission asymmetry (187) and angle of spin rotation (188) near p-wave resonance (a), near s-wave resonance (b), in a wide range of energy (c). Solid line — ϵ_P/n , dashed line — χ_P/n .

Fig.6. P-,T-noninvariant transmission asymmetry (187) and angle of spin rotation (188) near p-wave resonance (a), near s-wave resonance (b), in a wide range of energy (c). Solid line — ϵ_{PT}/n , dashed line — χ_{PT}/n .

Fig.7. P-noninvariant transmission asymmetry (187) and angle of spin rotation (188), as well as a sum of partial cross section $\sigma = \sigma_s + \sigma_p$ near p-wave resonance. Solid line — ϵ_P/n , dashed line — χ_P/n , dotted line — σ .

Fig.8. P-noninvariant transmission asymmetry (187) and angle of spin rotation (188), as well as a sum of partial cross section $\sigma = \sigma_s + \sigma_p$ near an interference minimum close to the s-wave resonance. Solid line — ϵ_P/n , dashed line — χ_P/n , dotted line — σ .

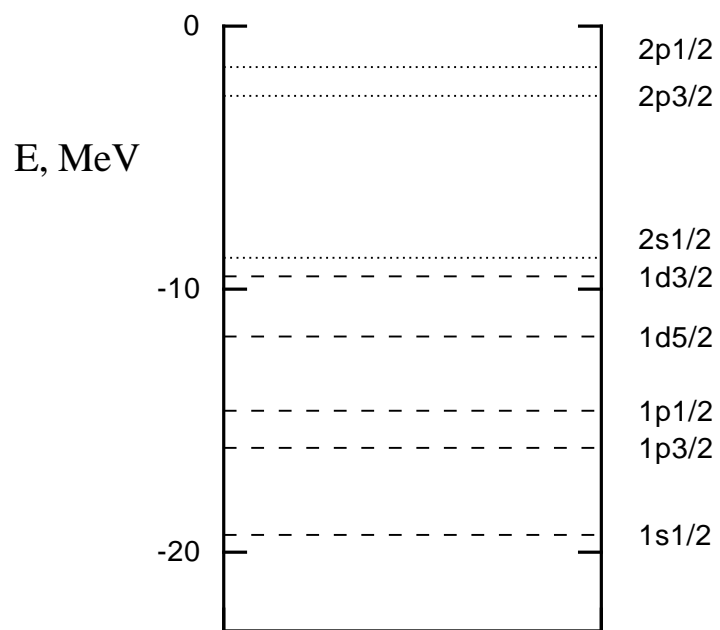


Fig.1

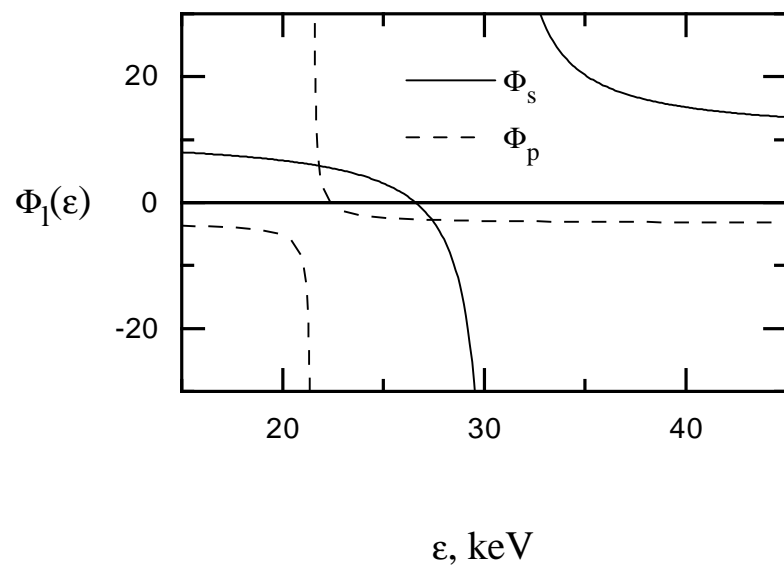


Fig.2

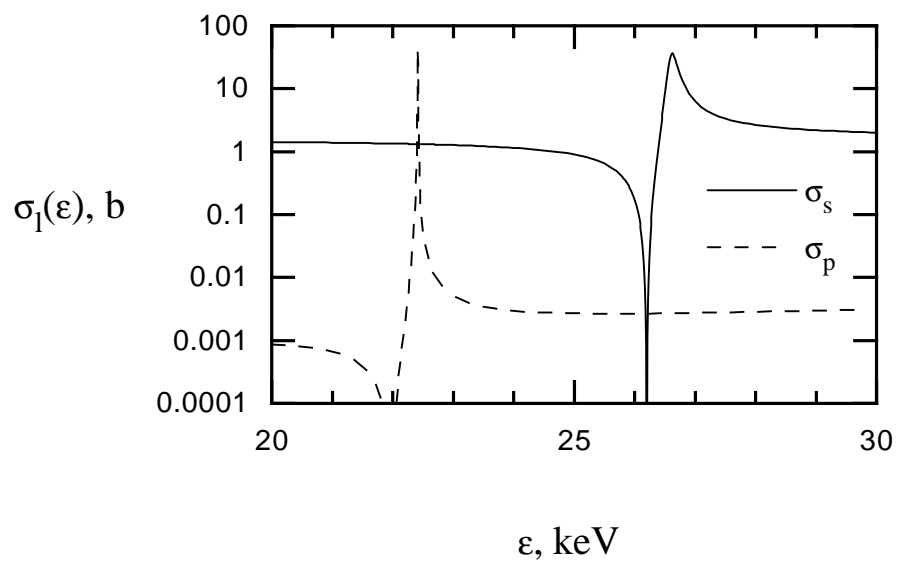


Fig.3

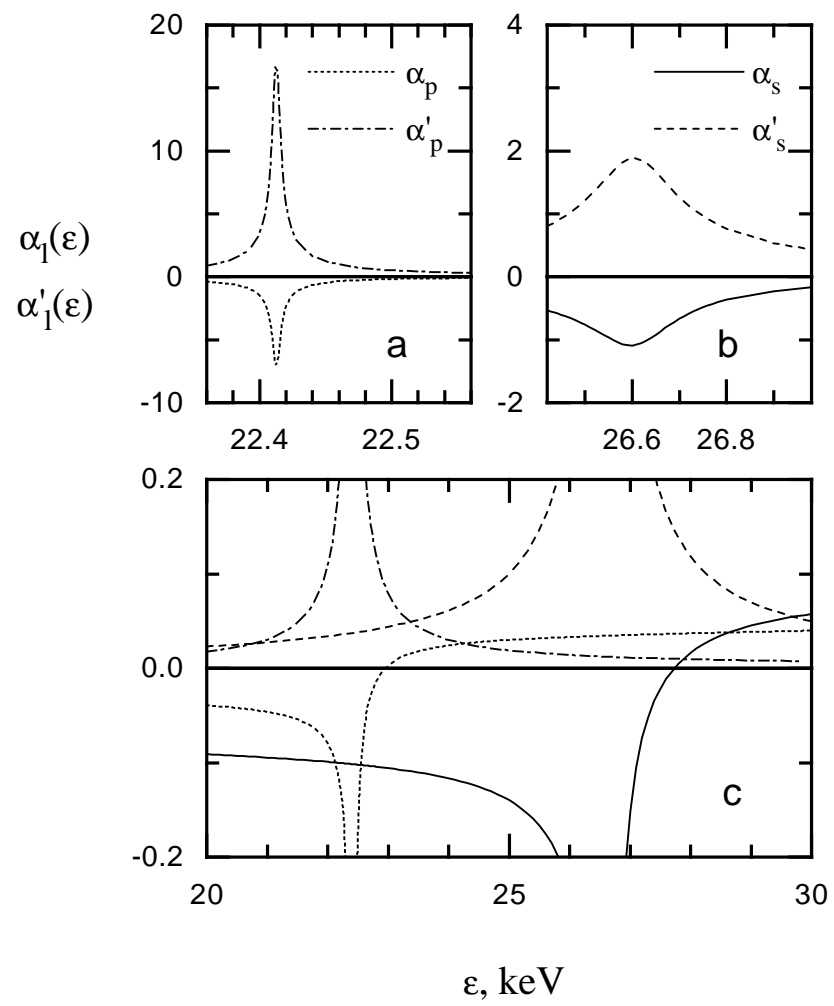


Fig.4

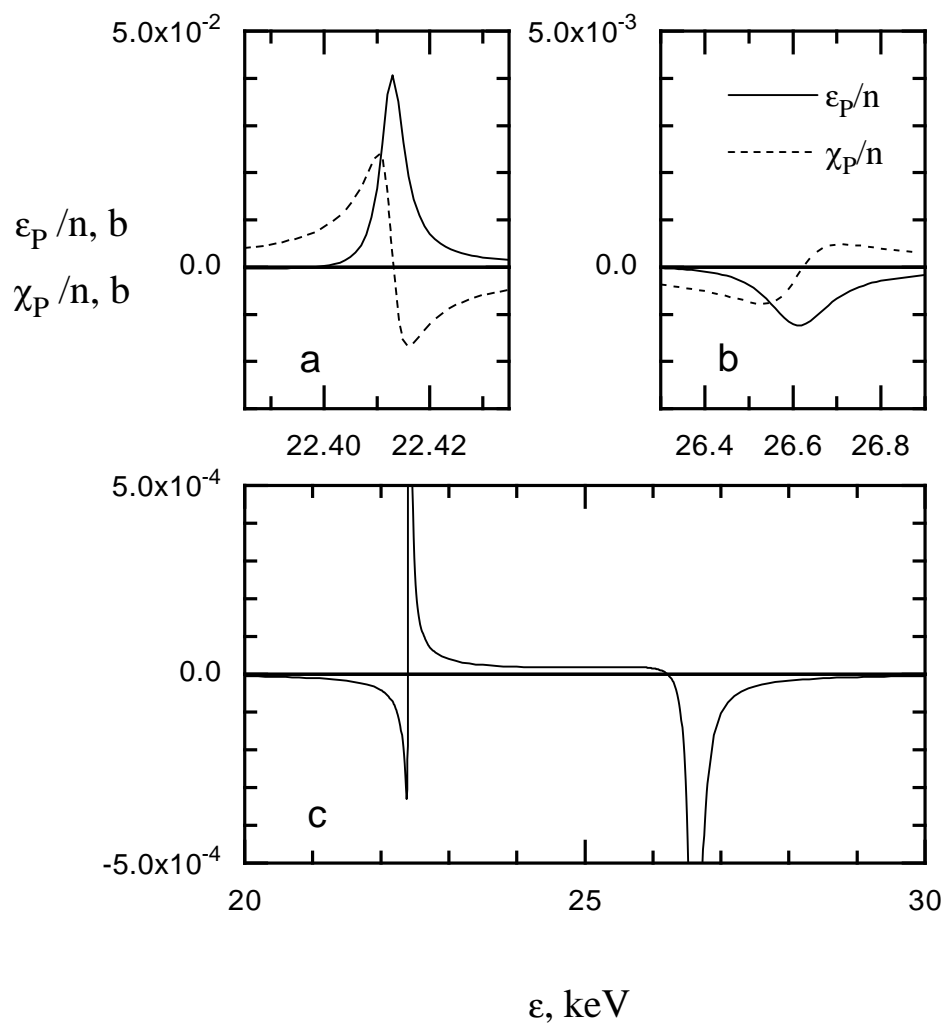


Fig.5

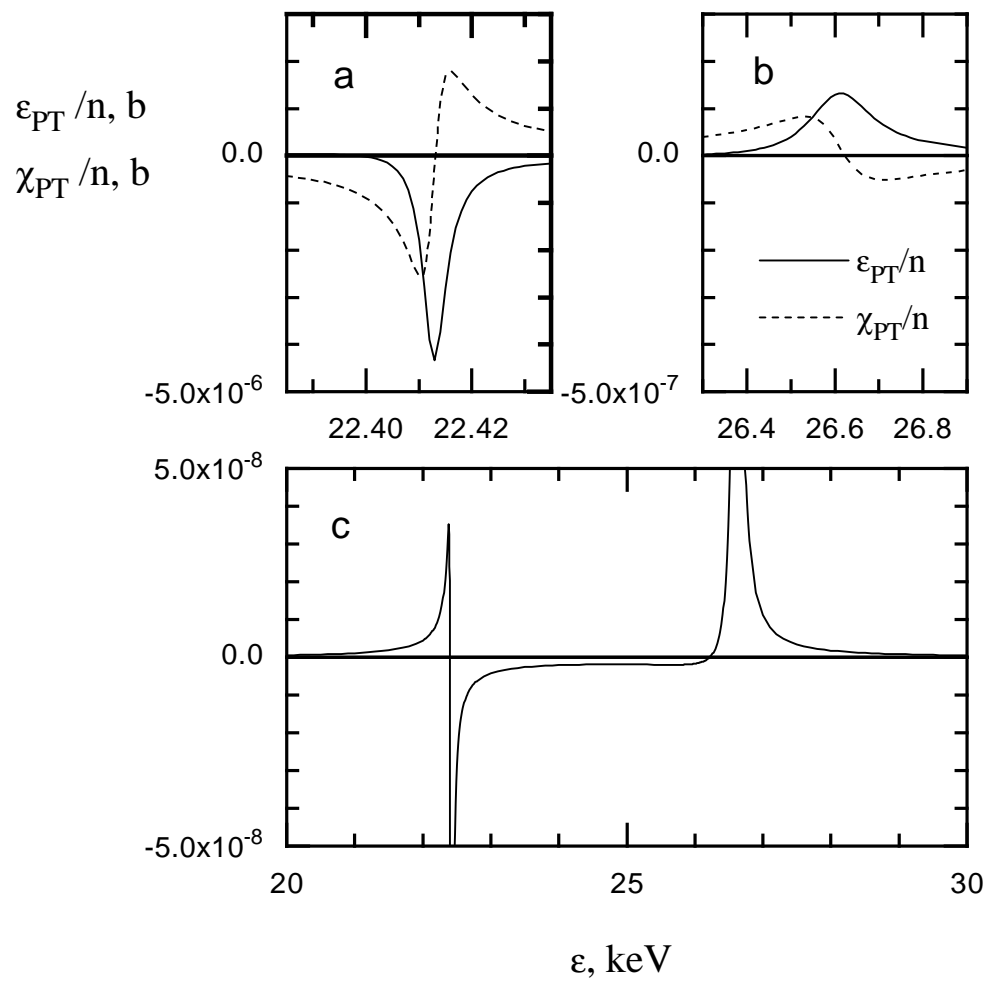


Fig.6

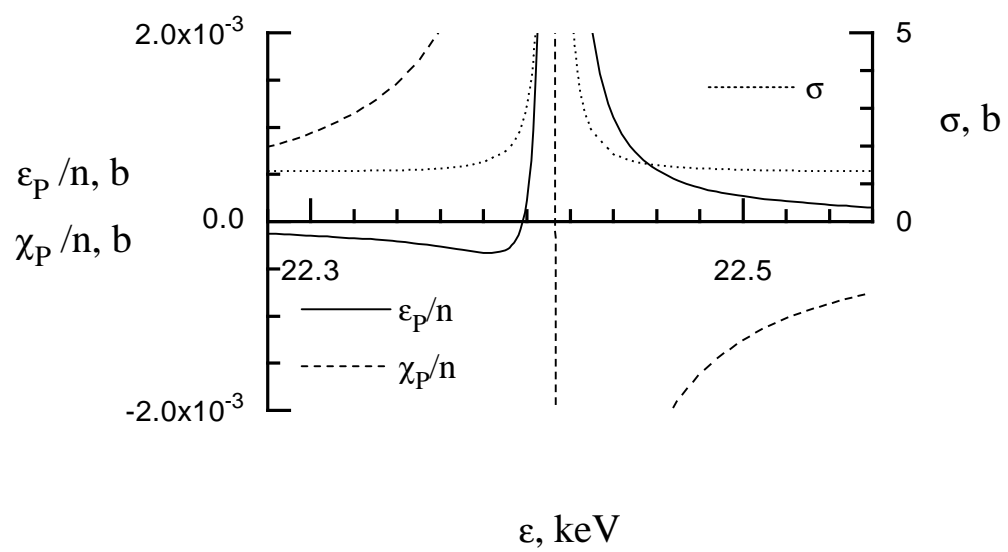


Fig.7

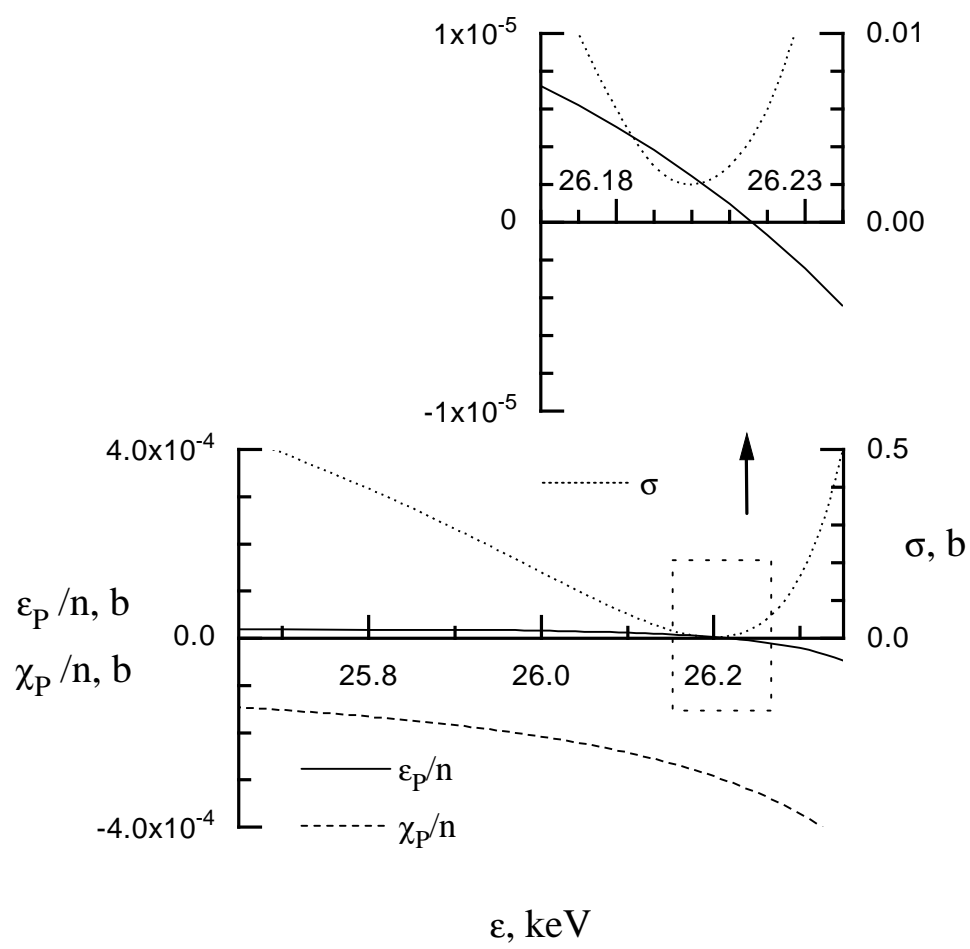


Fig.8